ACAP

Alabama Comprehensive Assessment Program

SUMMATIVE

Grade 2
Mathematics

Alabama Educator Instructional Supports

Alabama Course of Study Standards

DATA RECOGNITION DRC CORPORATION

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Introduction

The *Alabama Instructional Supports: Mathematics* is a companion to the 2019 *Alabama Course of Study: Mathematics* for Grades K–12. Instructional supports are foundational tools that educators may use to help students become independent learners as they build toward mastery of the *Alabama Course of Study* content standards. **Instructional supports are designed to help educators engage their students in exploring, explaining, and expanding their understanding of the content standards.**

The content standards contained within the course of study may be accessed on the Alabama State Department of Education (ALSDE) website: https://www.alabamaachieves.org/. When examining these instructional supports, educators are reminded that content standards indicate minimum content—what all students should know and be able to do by the end of each grade level or course. Local school systems may have additional instructional or achievement expectations and may provide instructional guidelines that address content sequence, review, and remediation.

The instructional supports are organized by standard. Each standard's instructional support includes a statement of the content standard, guiding questions with connections to mathematical practices, key academic terms, and additional resources.

Content Standards

The content standards are the statements from the 2019 *Alabama Course of Study: Mathematics* that define what all students should know and be able to do at the conclusion of a given grade level or course. Content standards contain minimum required content and complete the phrase "Students will ______."

Guiding Questions with Connections to Mathematical Practices

Guiding questions are designed to create a framework for the given standards and to engage students in exploring, explaining, and expanding their understanding of the content standards provided in the 2019 *Alabama Course of Study: Mathematics*. Therefore, each guiding question is written to help educators convey important concepts within the standard. By utilizing guiding questions, educators are engaging students in investigating, analyzing, and demonstrating knowledge of the underlying concepts reflected in the standard. An emphasis is placed on the integration of the eight Student for Mathematical Practices.

The Student for Mathematical Practices describe the varieties of expertise that mathematics educators at all levels should seek to develop in their students. They are based on the National Council of Teachers of Mathematics process standards and the strands of mathematical proficiency specified in the National Research Council's report *Adding It Up: Helping Children Learn Mathematics*.

The Student for Mathematical Practices are the same for all grade levels and are listed below.

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Each guiding question includes a representative set of sample activities and examples that can be used in the classroom. The set of activities and examples is not intended to include all the activities and examples that would be relevant to the standard.

Key Academic Terms

These academic terms are derived from the standards and are to be incorporated into instruction by the educator and used by the students.

Additional Resources

Additional resources are included that are aligned to the standard and may provide additional instructional support to help students build toward mastery of the designated standard. Please note that while every effort has been made to ensure all hyperlinks are working at the time of publication, web-based resources are impermanent and may be deleted, moved, or archived by the information owners at any time and without notice. Registration is not required to access the materials aligned to the specified standard. Some resources offer access to additional materials by asking educators to complete a registration. While the resources are publicly available, some websites may be blocked due to Internet restrictions put in place by a facility. Each facility's technology coordinator can assist educators in accessing any blocked content. Sites that use Adobe Flash may be difficult to access after December 31, 2020, unless users download additional programs that allow them to open SWF files outside their browsers.

Printing This Document

It is possible to use this entire document without printing it. However, if you would like to print this document, you do not have to print every page. First, identify the page ranges of the standards or domains that you would like to print. Then, in the print pop-up command screen, indicate which pages you would like to print.

1

Operations and Algebraic Thinking

Represent and solve problems involving addition and subtraction.

Note: Second grade problem types include adding to, taking from, putting together, taking apart, and comparing with unknowns in all positions.

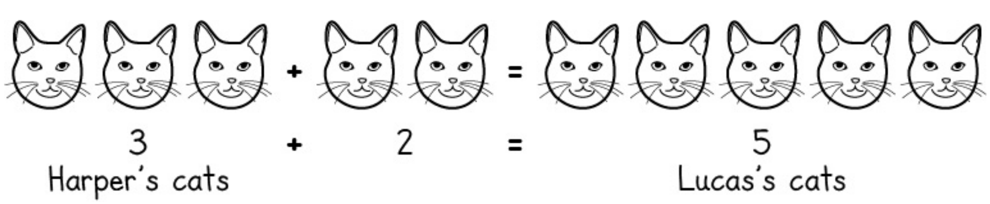
1. Use addition and subtraction within 100 to solve one- and two-step word problems by using drawings and equations with a symbol for the unknown number to represent the problem.

Guiding Questions with Connections to Mathematical Practices:

How can a word problem be represented in a variety of ways?

M.P.4. Model with mathematics. Represent all addition and subtraction situation types with manipulatives, drawings, or equations. For example, use blocks to represent a situation where some of the 24 students in a classroom leave the classroom to go to the media center, leaving 16 students in the classroom. That same situation could also be represented by the equation 24 – ☐ = 16, where ☐ equals the number of students that left the classroom to go to the media center.

- Ask students to solve addition and subtraction word problems and represent them through the use of manipulatives, drawings, or equations. Have students show or explain how to use a manipulative, drawing, or equation to help solve each word problem.
    - Harper has 3 cats. Lucas has 2 more cats than Harper. How many cats does Lucas have?



    - Madison has 17 bracelets and 13 rings. How many pieces of jewelry does she have in all?

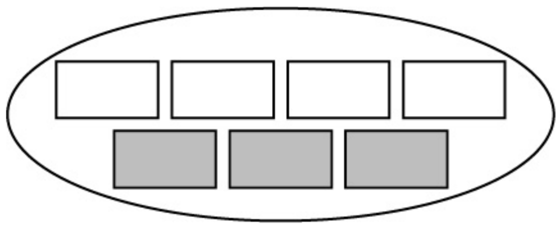
30

17 + 13 = ☐

- Ask students to write a word problem that could be represented by a given set of manipulatives, a drawing, and/or an equation.
    - Write a word problem that could be represented by the equation 14 – 6 = ☐.

      Alex has 14 baseball cards. She gives 6 away to her little sister. How many baseball cards does Alex have left?

    - Write a word problem that could be represented by the blocks shown.



There are 4 strawberries and 3 oranges in a bowl. How many total pieces of fruit are in the bowl?

How can an addition or subtraction equation be represented by a related equation to best represent a given situation?

M.P.7. Look for and make use of structure. Represent any addition or subtraction equation in other related equations by rearranging the addends and their placement in relation to the equal sign depending on context. For example, 13 + 9 = 22 is related to 9 + 13 = 22, 22 = 13 + 9, 22 = 9 + 13, 22 – 13 = 9, 9 = 22 – 13, 22 – 9 = 13, and 13 = 22 – 9. Additionally, sometimes certain forms of an equation lend themselves to the context of a situation better than others, whereas in other cases, many forms of an equation lend themselves equally to the context of a situation.

- Ask students to name the seven other equations that are related to a given equation. For example, write the seven other equations related to $31 + 6 = 37$.

$6 + 31 = 37$

$37 = 6 + 31$

$37 = 31 + 6$

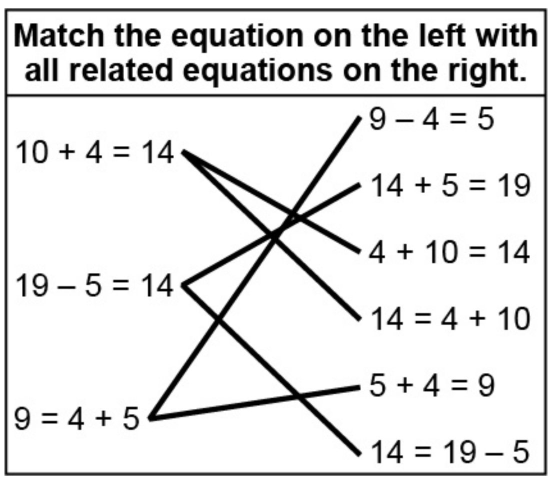
$37 - 6 = 31$

$37 - 31 = 6$

$31 = 37 - 6$

$6 = 37 - 31$

- Ask students to match related equations.



- Ask students to consider a given equation and then write and solve a related equation based on the context of the situation.
  - On Saturday, Max and Oliver ride their bikes. Max rides his bike for 3 hours. Oliver rides his bike for 1 hour less than Max does. The boys use the equation 3 – ☐ = 1 to help them figure out how long Oliver rides his bike on Saturday. What is another equation they could use to help them figure out how long Oliver rides his bike on Saturday?

    3 – 1 = ☐

    Oliver rides his bike for 2 hours on Saturday.
  - Layla has 8 pens and 9 pencils. She uses the equation ☐ = 8 + 9 to find out how many pens and pencils she has in all. What is another equation she could use?

    9 + 8 = ☐

    Layla has 17 writing utensils.

How can the context of a situation with an unknown number help to determine which operation(s) to use in a word problem?

M.P.2. Reason abstractly and quantitatively. Determine which operation to use in a word problem by making sense of the situation. For example, the situation "Mason has 19 dinosaur toys. Together, he and Rachel have 45 dinosaur toys. How many dinosaur toys does Rachel have?" can be represented by using either an addition equation, 19 + ☐ = 45, or a subtraction equation, 45 – 19 = ☐. Additionally, although there are choices as to which equation and operation to use when solving a word problem, oftentimes one equation is much easier to use than another.

- Ask students to read a number of word problems and to determine which operation, addition or subtraction, would be easier to use in order to solve the word problem. Possible student responses to the given word problems are shown.
  - Lila reads for 18 minutes before bed on Tuesday night and 20 minutes before bed on Wednesday night. How many total minutes does she spend reading before bed on Tuesday and Wednesday nights?

    addition

  - Anthony does homework for a total of 55 minutes on Monday and Thursday evenings. On Monday evening, he does homework for 27 minutes. How long does Anthony do homework on Thursday evening?

    subtraction

- Give students a word problem and an equation that could be used to represent it. Then, ask students to write and solve a related equation that is of the opposite sign that would be easier to use to solve the word problem.
  - Alexis has 4 more cousins than Jamie. Jamie has 8 cousins. How many cousins does Alexis have? Equation: $8 = \square - 4$

    $8 + 4 = \square$

    $\square = 12$

  - Some students were sitting at a lunch table. Later, 6 more students joined the table. Then, there were a total of 15 students sitting at the table. How many students were originally sitting at the lunch table? Equation: $\square + 6 = 15$

    $15 - 6 = \square$

    $\square = 9$

How does a two-step problem relate to a one-step problem?

M.P.6. Attend to precision. Interpret a two-step problem as a series of one-step problems. For example, the situation "Jude has 18 markers. He throws away 7 that were dried out before buying 10 more at the store. How many markers does Jude have now?" can be represented as 18 – 7 + 10 and is the same as first subtracting 7 from 18 and then adding 10 to the answer to find the total number of markers. Students often struggle with multi-step word problems, so attention to precision should be emphasized. Additionally, when breaking a two-step problem into separate one-step problems, sometimes the order in which the numbers are used to solve the problem matters and sometimes the order in which the numbers are used to solve the problem does not matter.

- Ask students to solve two-step word problems involving addition and/or subtraction by first separating each problem into two individual one-step problems.
    - There are 25 students in Mr. Johnson's class and 27 students in Mrs. Schmidt's class when no one is absent from either class. On Tuesday, there were 6 students absent between the two classes. How many total students were in Mr. Johnson's and Mrs. Schmidt's classes on Tuesday?

Problem 1: 25 + 27 = <u>52</u>

Problem 2: <u>52</u> – 6 = ☐

There were 46 total students on Tuesday.

- A gym teacher has a total of 39 sports balls for her students to use. She has 18 basketballs and 6 volleyballs. The rest of the balls are tennis balls. How many tennis balls does the gym teacher have?

Problem 1: 39 – 18 = 21

Problem 2: 21 – 6 = ☐

The teacher has 15 tennis balls.

OR

Problem 1: 18 + 6 = 24

Problem 2: 39 – 24 = ☐

The teacher has 15 tennis balls.

- Hunter collects bugs. He collects 4 bugs on Saturday, 9 bugs on Sunday, and 8 bugs on Monday. How many bugs does Hunter collect in all?

Problem 1: 8 + 9 = 17

Problem 2: 17 + 4 = ☐

Hunter collects 21 bugs.

OR

Problem 1: 4 + 9 = 13

Problem 2: 13 + 8 = ☐

Hunter collects 21 bugs.

- Ask students to use two given one-step equations to write a multi-step word problem.

$$12 + 9 = \_\_\_$$

and

$$\_\_\_ - 4 = \square$$

Casey has 12 dollars on Monday. He then gets paid 9 more dollars on Tuesday for doing his chores. Later in the week, he spends 4 dollars on ice cream. How much money does Casey have now?

Key Academic Terms:

add, subtract, compare, unknown, equation, equal addends, sum, minuend, subtrahend, difference, adding to, taking from, putting together, taking apart, comparing with unknowns

Additional Resources:

- Book: Davenport, L. R., Henry, C. S., Clements, D. H., & Sarama, J. (2019). *No more math fact frenzy*. Portsmouth, NH: Heinemann.
- Activity: Race to 100

2a

Operations and Algebraic Thinking
Add and subtract within 20. *Note: fluency vs. automaticity. Fluency involves a mixture of "just knowing" answers, knowing answers from patterns, and knowing answers from the use of strategies. The word fluently is used in the standards to mean accurately, efficiently and flexibly. Automaticity of facts becomes evident when a student no longer uses a pattern or mental algorithm to determine the answer.*
2. Fluently add and subtract within 20 using mental strategies such as counting on, making ten, decomposing a number leading to ten, using the relationship between addition and subtraction, and creating equivalent but easier or known sums. a. State automatically all sums of two one-digit numbers.

Guiding Questions with Connections to Mathematical Practices:

How can mental strategies help to add and subtract numbers?

M.P.7. Look for and make use of structure. Apply the use of mental strategies (counting on, making 10, decomposing a number leading to 10, doubling, relating addition to subtraction, creating equivalent sums) to add and subtract numbers. For example, $11 + 7$ is the same as $10 + 1 + 7$. Additionally, the use of mental strategies like these facilitate faster and easier methods for adding and subtracting numbers.

- Ask students to solve addition and subtraction equations using mental strategies and have the students show or explain how they used their strategies to solve the equation.
  - 6 + 2 = ☐

    I can start with 6 and add 1 to get 7, then add 1 more to get 8. Therefore, 6 + 2 = 8. (using counting on strategy)
  - 15 – 8 = ☐

    I can use the fact that 15 − 5 = 10 and then subtract the 3 from the 10 to get 7, because 5 and 3 add to 8. Therefore, 15 − 8 = 7. (using decomposing to 10)
  - 6 + 9 = ☐

    Because 9 is close to 10, I can mentally use a number line and start at 6, make a jump of 10 to 16, and then jump back one, because 10 − 1 = 9. Therefore, 6 + 9 = 15. (using creating equivalent sums)
- Ask students to study the way an addition or subtraction equation is being solved and then identify which strategy is being used to help solve it.
  - 4 + 9 = ☐

    4 + 6 + 3 = 10 + 3 = 13

    The 9 was partitioned into 6 and 3. Then, 10 from 4 + 6 was added to 3, so 10 + 3 = 13. (making 10)
  - 17 – 8 = ☐

    Since 8 + 9 = 17, then 17 − 8 must equal 9. (relating addition to subtraction)

What does adding or subtracting a zero mean?

M.P.2. Reason abstractly and quantitatively. Verify that adding does not always result in a larger number and subtracting does not always result in a smaller number, since adding or subtracting a zero does not change a number. For example, starting with 9 cookies and not giving any away, 9 – 0, results in the same number of cookies, 9. Additionally, starting with 4 hair ties and not buying any more at the store, $4 + 0 = 4$, results in the same number of hair ties.

- Ask students to solve a number of equations that involve adding or subtracting a zero. After students have completed the equations, ask them to record any observations they made. Students will see that when adding or subtracting a zero the result is always the non-zero number. For example, find the answers to the following equations.
    - $2 + 0 = \square$
    - $0 + 11 = \square$
    - $15 - 0 = \square$
    - $8 - 0 = \square$

  Adding or subtracting a zero means the other number is not changed.

  So the answers are 2, 11, 15, and 8.

- Ask students to write and solve equations for word problems involving adding or subtracting a zero.
    - There are 9 students sitting together at a lunch table. No other students join them at the table during lunch. How many total students are at the table at the end of lunch?

      $$9 + 0 = 9$$

    - Jamie orders 2 slices of pizza. Jamie does not eat any of the slices of pizza. How many slices of pizza does Jamie have left?

      $$2 - 0 = 2$$

Key Academic Terms:

add, subtract, mental strategies, counting on, making 10, decompose, equivalent sums, doubling

Additional Resources:

- Book: O'Connell, S., & SanGiovanni, J. (2015). *Mastering the basic math facts in addition and subtraction*. Portsmouth, NH: Heinemann.
- Book: Davenport, L. R., Henry, C. S., Clements, D. H., & Sarama, J. (2019). *No more math fact frenzy*. Portsmouth, NH: Heinemann.
- Article: Singapore strategies for teaching addition and subtraction
- Article: The path to automaticity with addition facts
- Activity: Ten frame
- Activity: Number fact families
- Game: Math Man Jr.
- Game: Math stack

3a

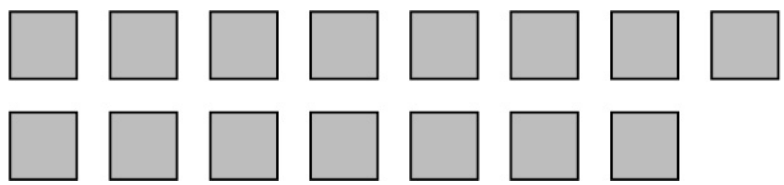
Operations and Algebraic Thinking
Work with equal groups of objects to gain foundations for multiplication.
3. Use concrete objects to determine whether a group of up to 20 objects is even or odd. a. Write an equation to express an even number as a sum of two equal addends.

Guiding Questions with Connections to Mathematical Practices:

How can it be determined whether a number is even or odd?

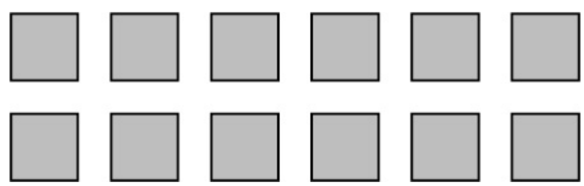
M.P.7. Look for and make use of structure. Organize groups of objects in a way that pairs them up, matches from one row to another, or groups them into two equal groups to determine whether a number is even or odd. For example, 17 pennies laid out in two rows on a desk, organized so that the rows line up with each other, will have one row that is one penny longer than the other row; therefore, 17 is an odd number. However, 18 pennies laid out in two rows on a desk, organized so that the rows line up with each other, will have two rows with the same number of pennies in each row; therefore, 18 is an even number.

- Ask students to use a drawing or a manipulative (such as blocks, tokens, or pennies) to determine whether a given number is even or odd. Students do this by attempting to arrange a given number of manipulatives into two equal rows. If the rows align and are equal, then the given number is even, and if one row has one more of the manipulative in it than the other row, then the given number is odd.
    - 15



odd

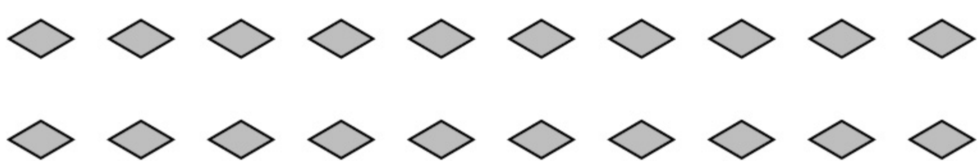
    - 12



even

- Ask students to look at a series of numbers that are each being modeled by two rows of an object and use the model to tell whether the number is even or odd.

odd



even

How does writing an equation help to compose an even or odd number?

M.P.2. Reason abstractly and quantitatively. Demonstrate any even number as the sum of two equal addends. For example, $12 = 6 + 6$, therefore 12 is an even number. However, 13 is an odd number because, although there is a pair of consecutive numbers that will add together to make a sum of 13, there is not a whole number that can be added to itself to make a sum of 13. Additionally, all odd numbers can be written as the sum of two equal numbers plus 1, so 13 is an odd number that can be written as $6 + 6 + 1 = 13$.

- Ask students to write a given even number as the sum of two equal whole numbers.
  - 6

    6 = 3 + 3
  - 18

    18 = 9 + 9
  - 8

    8 = 4 + 4

- Ask students to write a given odd number as the sum of two equal whole numbers plus 1 or as the sum of two consecutive whole numbers.
    - 17

      17 = 8 + 9

      OR

      17 = 8 + 8 + 1
    - 7

      7 = 3 + 4

      OR

      7 = 3 + 3 + 1
    - 11

      11 = 5 + 6

      OR

      11 = 5 + 5 + 1

- Ask students to look at a series of addition expressions to determine whether the sum of the expression will be an even or odd number, without solving the expression. All expressions need to be either equal addends, equal addends plus 1, or two consecutive addends.

$2 + 2$	even
$4 + 5$	odd
$8 + 7$	odd
$5 + 5$	even
$9 + 9 + 1$	odd

Key Academic Terms:

add, subtract, even, odd, equation, equal addends, whole number, pair, match, sum

Additional Resources:

- Lesson: Odd and even numbers
- Book: Cristaldi, K. (1996). *Even Steven and odd Todd*. New York, NY: Cartwheel Books. Activity
- Game: Odd or even

4a

Operations and Algebraic Thinking
Work with equal groups of objects to gain foundations for multiplication.
4. Using concrete and pictorial representations and repeated addition, determine the total number of objects in a rectangular array with up to 5 rows and up to 5 columns. a. Write an equation to express the total number of objects in a rectangular array with up to 5 rows and up to 5 columns as a sum of equal addends.

Guiding Questions with Connections to Mathematical Practices:

How can arrays help with solving addition problems?

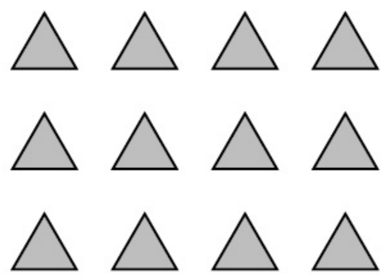
M.P.7. Look for and make use of structure. Interpret a rectangular array as a display of objects relating to the numbers in an addition problem. For example, an array that has 5 rows and 3 columns of objects can represent $3 + 3 + 3 + 3 + 3$ or $5 + 5 + 5$. Use skip-counting to find the total. Skip-counting to find the total number of objects in an array involves counting by the number of rows in the array and repeating that count for each column in the array, or vice versa.

- Ask students to write an addition problem represented by the rows or the columns of an array.

addition problem: 4 + 4

OR

2 + 2 + 2 + 2

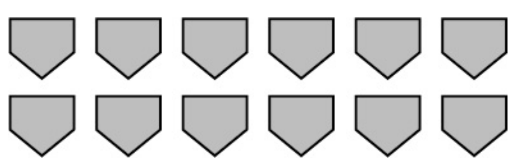


addition problem: 3 + 3 + 3 + 3

OR

4 + 4 + 4

- Ask students to draw an array to represent a repeated addition problem. For example, ask students to draw an array to represent the problem $2 + 2 + 2 + 2 + 2 + 2$. One possible response is shown.



Further, ask students to explain why an array cannot be drawn for a problem like $3 + 3 + 3 + 5$. In this case, the addition can be illustrated visually, but the pictures would not be in a rectangular array because not all the addends are the same.

How are rows and columns in an array related to addition equations?

M.P.8. Look for and express regularity in repeated reasoning. Know that an array has rows and columns that represent addends in an addition equation and can be interpreted in different ways. For example, an array that has 4 rows and 5 columns can be represented as $4 + 4 + 4 + 4 + 4 = 20$ or $5 + 5 + 5 + 5 = 20$. Additionally, if an array has the same number of rows and columns, there is only one addition equation to represent that array.

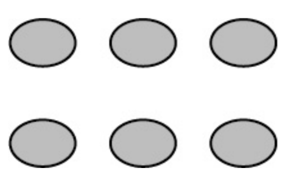
- Ask students to use an array and an addition equation that is represented by that array to name an additional addition equation that could also be represented by the array.

5 + 5 + 5 = 15

3 + 3 + 3 + 3 + 3 = 15

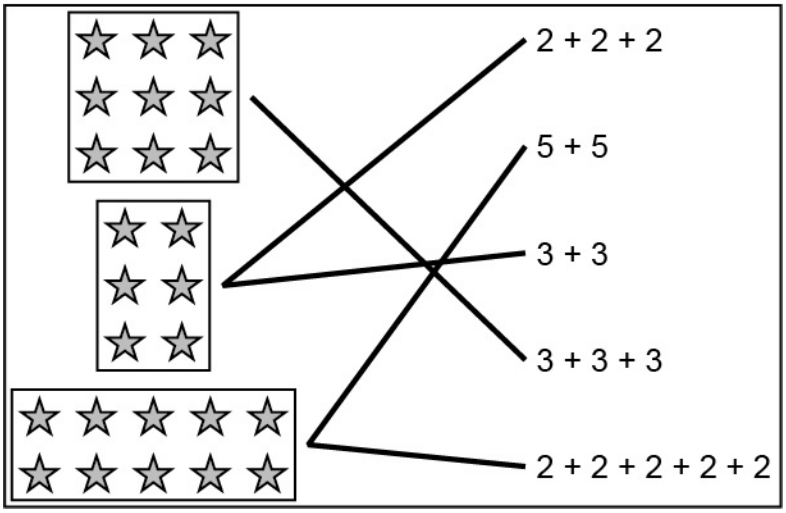
- Ask students to study an array and identify which number or numbers they could skip-count by to tell how many total objects are in the array. Then, ask students to tell the total number of objects that are in the array.

skip-count by 2, total of 4 objects



skip-count by 2 or 3, total of 6 objects

- Ask students to match an array to all the addition problems that it represents.



Key Academic Terms:

add, array, row, column, equation, addend, skip-counting, sum, total, repeated addition

Additional Resources:

- Article: How to teach multiplication
- Video: Skip-counting
- Book: Pinczes, E. J. (1993). *One hundred hungry ants*. Boston, MA: Houghton Mifflin Harcourt Books for Young Readers. Activity
- Activity: Roll a rectangular array
- Article: How to teach arrays
- Game: Multiplication Mine Jr.

5

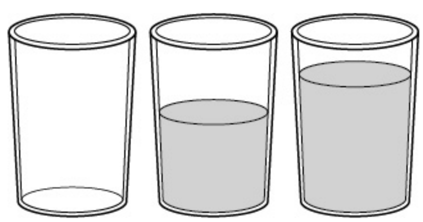
Operations and Algebraic Thinking
Understand simple patterns.
5. Reproduce, extend, create, and describe patterns and sequences using a variety of materials.

Guiding Questions with Connections to Mathematical Practices:

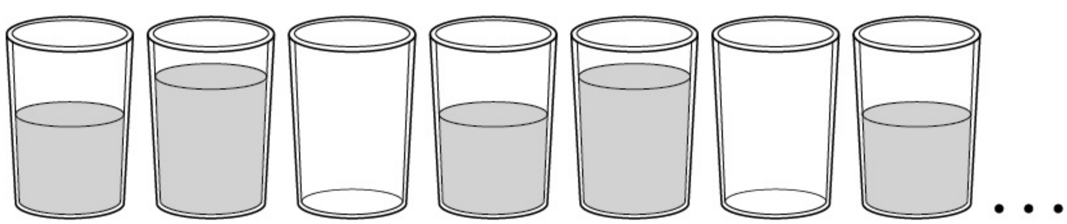
How can sounds and motions be used to represent patterns?

M.P.7. Look for and make use of structure. Identify which sound or sounds come next when given an acoustic pattern. For example, the seventh sound in the pattern *snap-snap-clap-snap-snap-clap* and so on will be *snap.* Additionally, the ninth and tenth hand gestures in the pattern *paper-scissors-rock-rock-paper-scissors-rock-rock* will be *paper-scissors*.

- Provide students with a pattern of sounds by tapping with a pencil on glasses that contain different amounts of water. Ask them to identify which glass should be tapped next to continue the pattern. For example, a pattern of sounds can be created by tapping on three glasses similar to the ones shown.

  Tap on the glasses in the following order.

  Students determine that the full glass of water should be tapped next to continue the pattern.

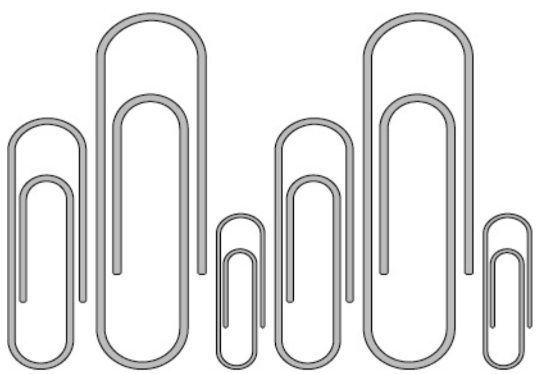
- Provide students with a pattern of arm motions. Ask them to make the arm motion that should come next to continue the pattern. For example, make the following arm motions in this order.

Students indicate the next arm motion in the pattern by raising both arms into the air.

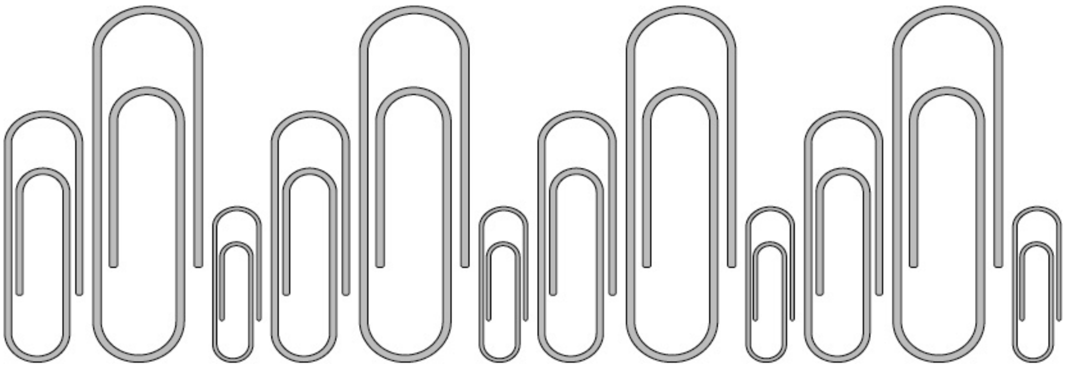
How can a collection of physical objects be used to form or continue patterns?

M.P.7. Look for and make use of structure. Select and order objects that continue a pattern until a particular number of objects is reached. For example, the eleventh bead in a necklace that begins with the pattern *red-white-pink-red-white-pink* will be *white*. Additionally, determine the number of a particular object when given the total number of objects within a pattern. For example, there will be 8 green beads in a 16-bead bracelet that follows the pattern *blue-green-green-yellow-blue-green-green-yellow* and so on.

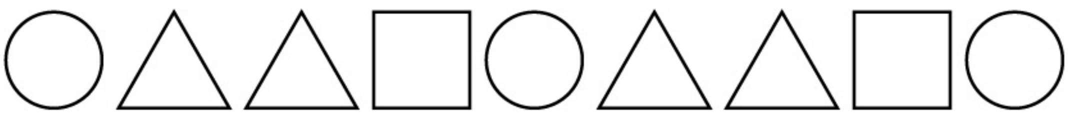
- Provide students with a collection of small, medium, and large paper clips. Ask them to extend a particular pattern of paper clips until the total number of clips is doubled. For example, show students the following pattern of 6 clips.



Ask students to extend the pattern until there are 12 clips. The completed pattern will look like this.



- Provide students with a pattern that is composed of three or more different shapes. Ask them to determine how many of each type of shape there will be when the pattern is extended to a particular total number of shapes. For example, show students the following pattern.



Then ask students to extend the pattern until there are 13 total shapes and count the number of each type of shape. In this case, the extended pattern will have 4 circles, 6 triangles, and 3 squares.

Key Academic Terms:

pattern, sequence

Additional Resources:

- Activity: Crack hackers safe
- Activity: Pattern blocks

6a

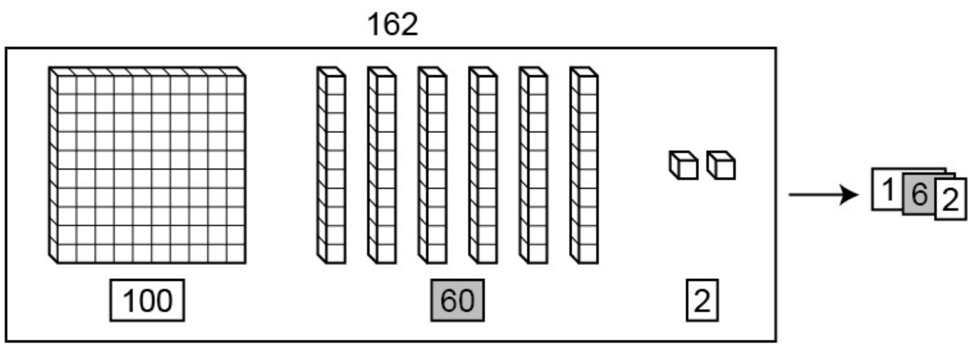
Operations with Numbers: Base Ten
Understand place value.
6. Explain that the three digits of a three-digit number represent amounts of hundreds, tens, and ones. a. Explain the following three-digit numbers as special cases: 100 can be thought of as a bundle of ten tens, called a "hundred," and the numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight, or nine hundreds (and 0 tens and 0 ones).

Guiding Questions with Connections to Mathematical Practices:

How can place value understanding help to represent a three-digit number?

M.P.7. Look for and make use of structure. Know the specific words used to identify the place values of digits, such as hundreds, tens, and ones, and use the words to represent numbers. For example, the number 278 can be represented as 2 hundreds, 7 tens, and 8 ones, as 27 tens and 8 ones, or as 278 ones. Additionally, the value of each digit in the number is determined by its place in the number, so the value of the digit 4 in 423 is 400, the value of the digit 2 is 20, and the value of the digit 3 is 3. Further, 100602 is NOT an accurate representation of one hundred sixty-two.

- Ask students to build numbers using base-ten materials and sets of numbered cards. The set of cards can include one card for each hundreds place (100–900), one card for each tens place (10–90), and one card for each ones place (1–9). Represent three-digit numbers using base-ten materials, such as base-ten blocks. Position the base-ten blocks for the number (for example, 162), and place the matching number cards (100, 60, and 2) below the corresponding base-ten blocks. Then, start with the hundreds card and place the tens and ones cards on top, keeping the cards right-aligned so each place digit is visible. Discuss how the number is built and the value of each digit in the number while drawing and recording the process.

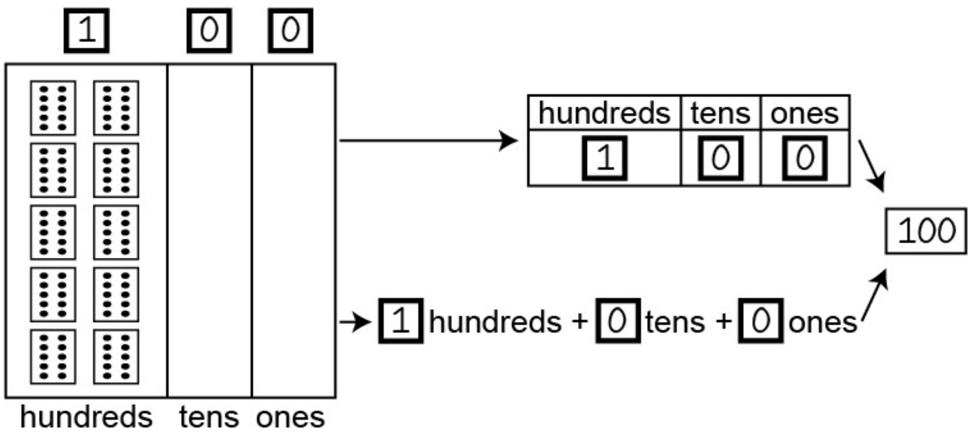


- Ask students to arrange a quantity of objects (between 150 and 999 objects) into groups of 10. Next, discuss how to use the groups of ten to find the total number of objects and how to make new groups from the groups of ten. When all objects are grouped, count the hundreds, the tens, and the ones separately. Then, have students record the total for each place, such as "3 hundreds + 9 tens + 5 ones."

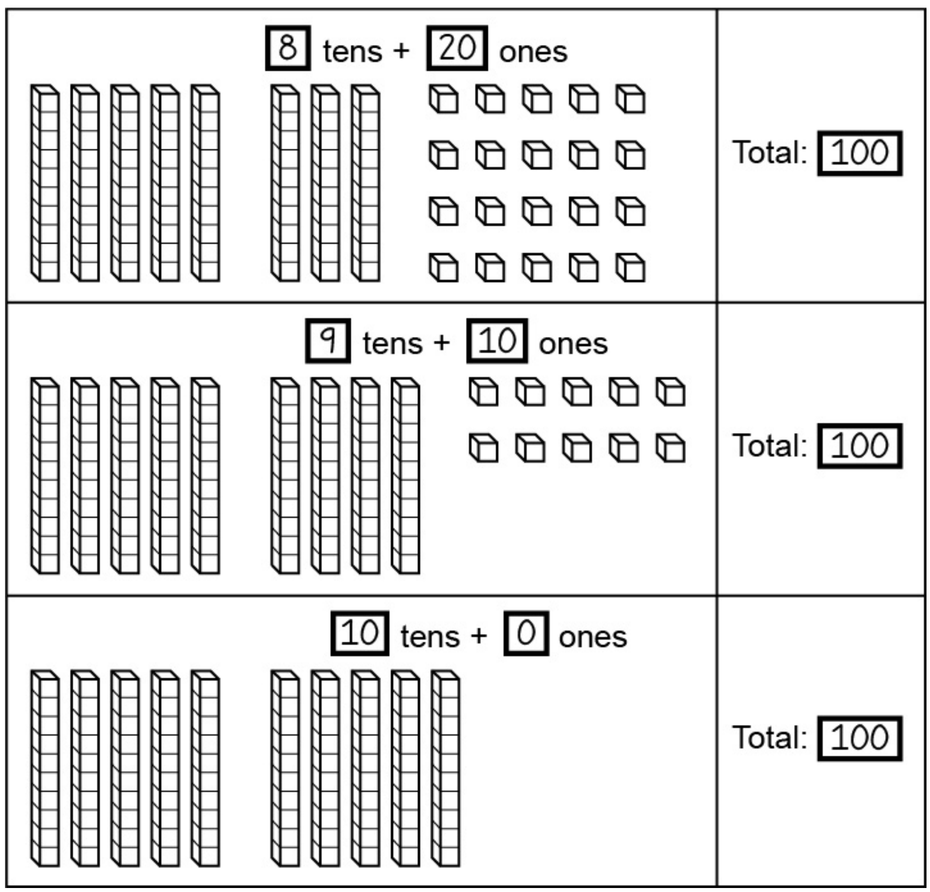
How does a bundle of 10 tens relate to a bundle of 10 ones?

M.P.8. Look for and express regularity in repeated reasoning. Relate the previous understanding that a bundle of 10 ones makes 1 ten to the idea of a bundle of 10 tens makes 1 hundred and explain how that relates to place values. For example, use base-ten blocks to make a ten (a rod) and put 10 tens together to make a hundred (a flat), and then write each of the numbers (1 one, 1 ten, and 1 hundred) in numerical form to see the place value of each. Additionally, create a group of 100 objects that is partly pre-grouped in sets of ten, count the objects, and then change the groupings without changing the total number of objects. Further, describe one hundred as 100 single objects (ones), as 10 groups of ten (tens), and as a singular group of a hundred.

- Ask students to use base-ten materials (such as cubes, objects in bags or containers, bundles of sticks or straws, or base-ten blocks) to count by ones, by groups, and by tens. The students first make groups of 10 objects. Note that a group of 10 objects is the same as 10 single objects. Then, count and group the groups of ten. For example, have students put 10 beads into small cups, and then put 10 of the small cups into a shoe box to make 100. Notice that the model for ten is 10 times as large as the model for one. Similarly, also notice that the model for one hundred is 10 times as large as the model for ten. The model shows one hundred as 1 group of a hundred, as 10 groups of ten, and as 100 single objects. Connect the model to the structure of the base-ten number system and place value by drawing the groups of ten, recording numerals in the indicated boxes, and writing the number in standard form.



- Given a set of objects that is partly grouped, show a collection of 100 objects that has pre-grouped sets of ten, and ask students to count the sets of ten and the number of single objects using any method. For example, show 8 groups of cubes that are connected in towers of 10 cubes, along with 20 unconnected cubes. Count the total number of cubes and share the methods used. Then, draw and record the total number of cubes. Next, change the groupings (make another tower of ten from the single cubes or break a tower of ten into single cubes) without changing the total number of cubes. Ask students to repeat the process of counting, sharing, drawing, and recording the total number of cubes. Have students repeat this process with different groupings and create other groupings.



When students are comfortable recognizing 10 groups of ten as one hundred, repeat this activity with three-digit numbers other than 100. The groupings should be changed several times without changing the total number of cubes. Discuss what can be done with excess or leftover groups of ten and single cubes. (Leftover groups of ten are recorded in the tens place and leftover single cubes are recorded in the ones place.)

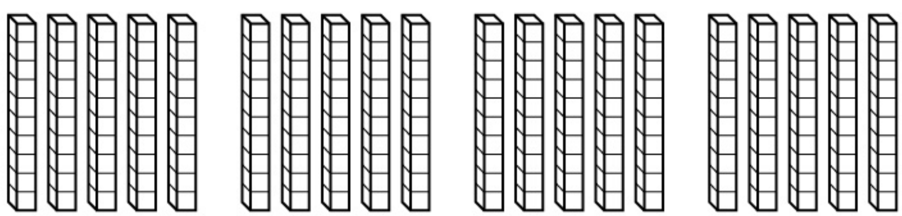
How can hundreds be composed to make multiple hundreds?

M.P.7. Look for and make use of structure. Construct bundles of hundreds to compose a given number of hundreds. For example, a bundle of 5 hundreds makes 500. Additionally, the number 300 can be represented as 3 hundreds, 0 tens, and 0 ones.

- Ask students to arrange a large quantity of objects (a multiple of 100, up to 900) into groups of 10 objects. Discuss how to use the groups of tens to find the total number of objects and how to make new groups from the groups of tens. When all the objects are grouped, count the groups of hundreds and determine that there are 0 groups of ten and 0 ones left over. Repeat this with another multiple of 100 objects. Identify that there are always 0 groups of ten and 0 ones left over when counting multiples of 100.

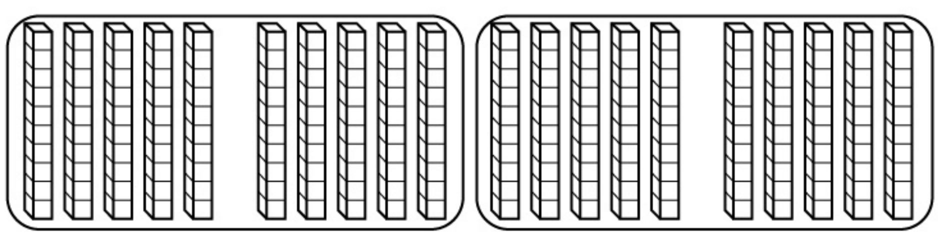
- Ask students to use base-ten blocks to represent a multiple of 100 using only tens. For example, use 20 tens to draw a representation of 200. Then, circle groups of 10 tens to make 2 groups of 1 hundred each and draw the new representation of 200. Ask students to repeat this with different multiples of 100.

Number
200
Tens Representation

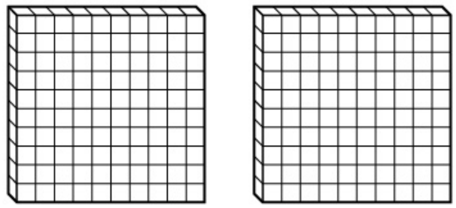


20 tens = 200

Hundreds Groups



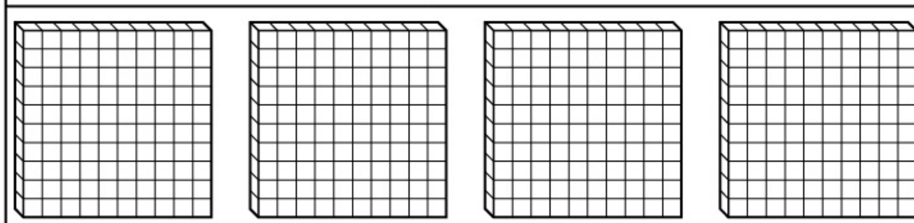
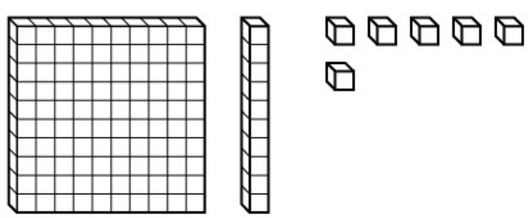
20 tens = 2 hundreds

Hundreds Representation
 2 hundreds = 200

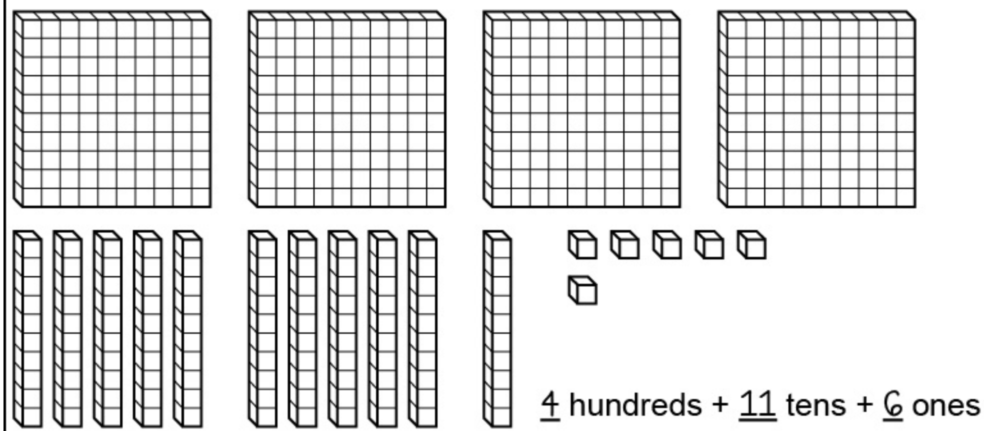
How can three-digit numbers be represented in different ways?

M.P.2. Reason abstractly and quantitatively. Verify that a number with three digits can be decomposed in multiple ways. For example, 461 can be decomposed to 4 hundreds, 6 tens, and 1 one; 46 tens and 1 one; 461 ones; or 3 hundreds and 161 ones; etc. Additionally, sets of hundreds, tens, and ones can be regrouped within a number to decompose the number in different ways. Further, the quantity does not change when a number is decomposed in different ways.

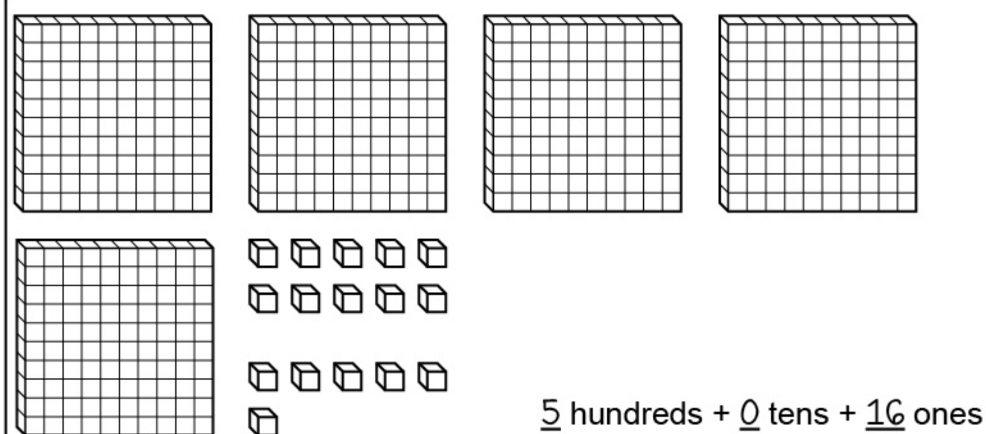
- Ask students to show a three-digit number with base-ten blocks using the standard representation for each place. Draw and record the representation. Then, find and record other ways to represent the same number.

Number
516
Representation
  5 hundreds + 1 ten + 6 ones

Representation



Representation

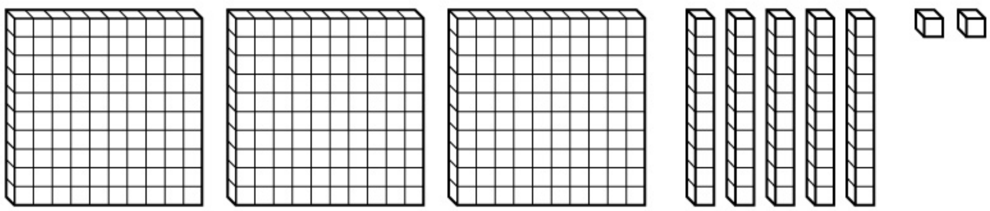
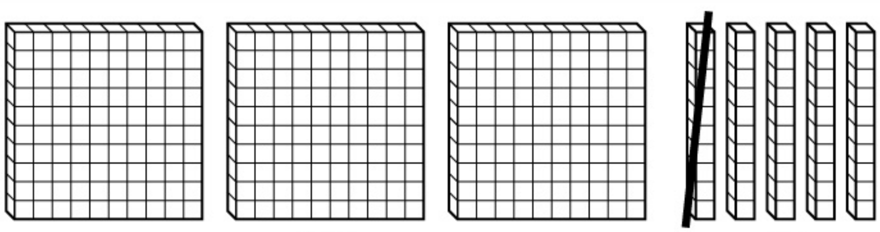
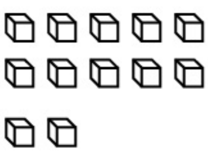


- Ask students to create and solve "Guess My Number" puzzles using base-ten materials as in the following examples:
    - My number has 2 hundreds, 14 tens, and 3 ones. What is my number?
    - My number has 14 tens, 1 hundred, and 42 ones. What is my number?

  Some more difficult puzzle examples could include the following:
    - My number is 358 when put with 2 more tens. What is my number?
    - My number has 29 ones, and it is between 50 and 60. What is my number? How many tens does my number have?

  Use pre-grouped base-ten models, such as bundles of popsicle sticks (pre-bundled into 10 groups of ten or 1 hundred in groups of ten and/or single sticks), to build the number given in the puzzle. Draw and record the solutions to the number puzzles and have students share their methods. Then, have students create other number puzzles to solve.

- Ask students to show the regrouping of a hundred or a ten within a three-digit number to decompose the number in a different way. Use base-ten blocks to represent a three-digit number using the standard representation for each place and draw the representation. Then, regroup 1 ten into 10 ones and record the second representation that is created by crossing out 1 ten and adding 10 ones to the ones column.

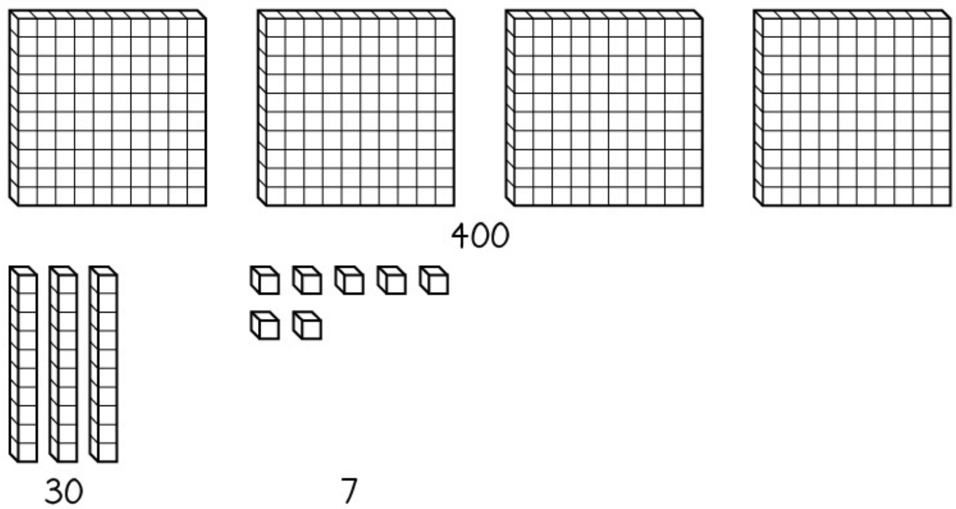
Number
352
Representation
 300 50 2
Regrouping
 300 40  12
Same Quantity
300 40 + 12 352

- Ask students to represent another number with base-ten blocks. Then regroup 1 hundred into 10 tens, recording the regrouping in the same way.

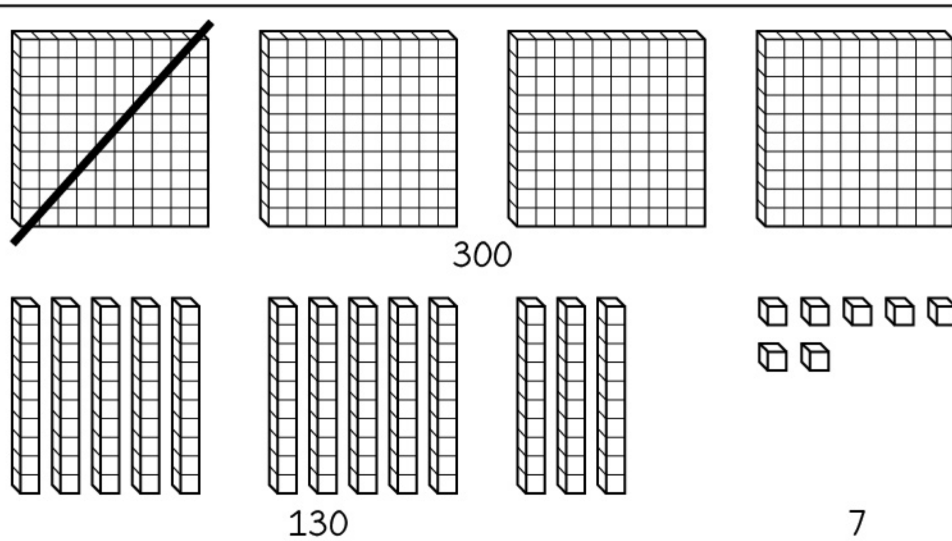
Number

437

Representation



Representation



Same Quantity

$$\begin{array}{r} 300 \\ 130 \\ +\quad 7 \\ \hline 437 \end{array}$$

Key Academic Terms:

digit, place value, hundreds, tens, ones, bundle, three-digit number, compose, decompose

Additional Resources:

- Article: 3 super tips for teaching place value
- Video: Place value: trading tens for hundreds
- Worksheets: Place value for 2nd grade
- Activity: Race to 100
- Video: Skip counting by 100
- Video: Counting by 100s song
- Lesson: Skip-count by 10s and 100s
- Article: Skip-counting
- Activity: Counting collections

7

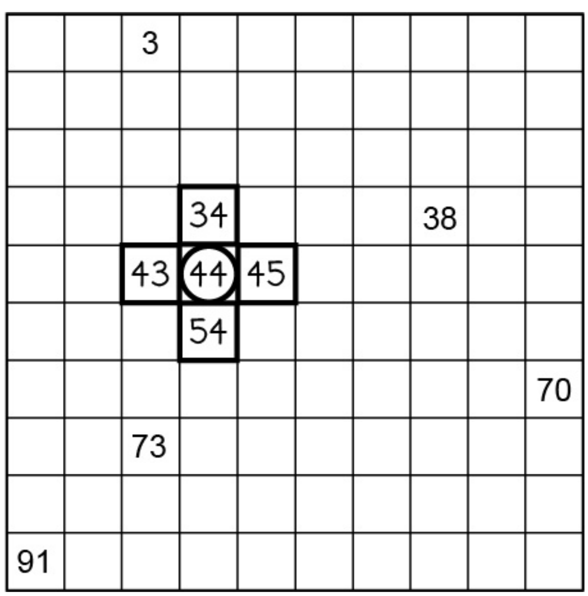
Operations with Numbers: Base Ten
Understand place value.
7. Count within 1000 by ones, fives, tens, and hundreds.

Guiding Questions with Connections to Mathematical Practices:

How can place value understanding help with counting?

M.P.7. Look for and make use of structure. Use place value understanding to relate counting single- and double-digit numbers to counting numbers with three digits. For example, the next three numbers when counting from 19 are 20, 21, and 22, and that relates to the next three numbers when counting from 819: 820, 821, and 822. Additionally, there are patterns to the way that numbers are formed, and the 0-to-9 sequence appears repeatedly in each place in the number system. Further, the sequence of numbers is related to number relationships.

- Ask students to find missing numbers and the neighbors to those missing numbers on a 100s chart. Students are shown a 100s chart with only a few numbers filled in. This can be an interactive chart on which number cards are placed, a projection shown to the whole class, or a worksheet given to groups, pairs, or individual students. A missing number is circled. Then find the missing number and the numbers directly to the left, to the right, above, and below the missing number.



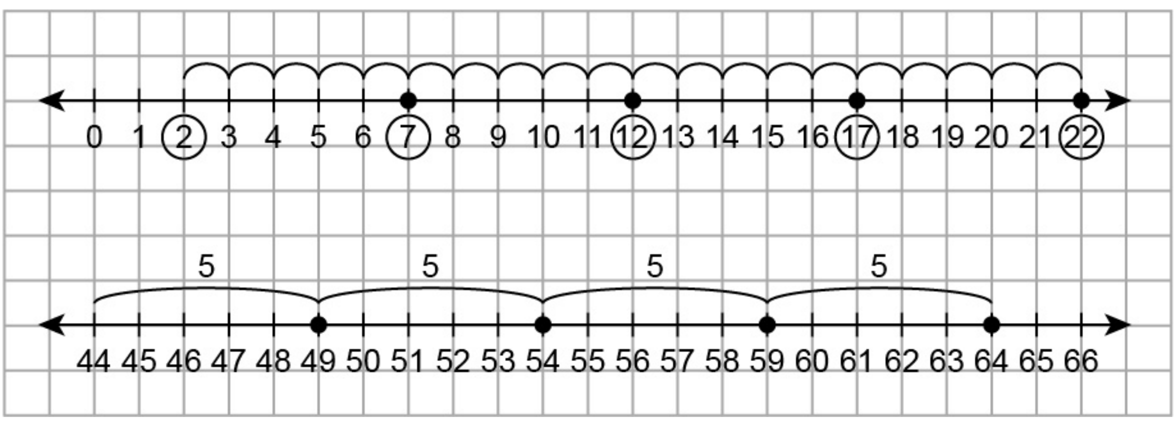
Ask students to discuss how they found the circled number and its neighbors and what they notice about the neighboring numbers, including diagonal neighbors. Extend this activity to the 1000s chart.

- Ask students to create a 1-to-1000 chart by taping ten 100s charts together in a long row or column. Discuss the process used to create the charts, how numbers change as they count from one hundred to the next hundred, and other patterns between the single-digit, two-digit, and three-digit numbers used.

What patterns emerge when skip-counting by 5s?

M.P.8. Look for and express regularity in repeated reasoning. Observe that skip-counting by 5s leads to a pattern in the ones place of alternating numbers and a pattern in the tens place of increasing by one every other time. For example, skip-counting by 5s starting at 2 results in 2, 7, 12, 17, 22, 27, and so on. Additionally, when skip-counting by 5s, 5 is added to each number to get the next number in the pattern. Further, skip-counting by 5s is related to skip-counting by 10s.

- Ask students to show skip-counting by 5s on a number line given a starting number. For example, start with 2 or 44, as shown.



Discuss the patterns in the numbers circled or plotted. The students may also discuss other numerical patterns in numbers that neighbor the circled or plotted numbers.

- Ask students to solve and create “Guess My Number” puzzles using 5s as in the following examples:
    - My number is the number you land on when you start at 20 and skip-count by 5s four times. What is my number?
    - My number is the number you land on when you start at 88 and skip-count by 5s three times. What is my number?

  Some more difficult puzzle examples could include the following:
    - My number is the number you land on when you start at 45 and skip-count backward by 5s two times. What is my number?
    - My number is the number you land on when you start at 64, skip-count forward by 5s five times, and then skip-count backward by 5s two times. What is my number?

  Predict the answer and use number lines or hundreds charts to help solve the puzzles. Next, draw and record the process used to solve the puzzles. Create other skip-counting puzzles for students to solve. Ask students to discuss how they made their initial predictions, the process used to solve the puzzles, and any patterns found.

What patterns emerge when skip-counting by 10s and 100s?

M.P.8. Look for and express regularity in repeated reasoning. Observe that skip-counting by 10s leaves the digit in the ones place the same while the digit in the tens place goes up by one each time. When skip-counting by 100s, both the ones and tens digits remain the same while the digit in the hundreds place goes up by 1 each time. For example, skip-counting by 100s starting at 47 results in 47, 147, 247, 347, and so on. Additionally, using these patterns to skip-count by 10s can be more efficient than counting by 1s or skip-counting by 5s. Further, using the same patterns to skip-count by 100s can be more efficient than skip-counting by 10s.

- Ask students to identify missing numbers on a 100s chart on which some of the numbers have been removed. In the first phase, the students are shown a 100s chart where a random selection of individual numbers have been removed. This can be a chart made of transparent pockets in which number cards can be placed or it can be projected while using objects such as cards or counters to cover some of the numbers. Ask students to replace the missing numbers.

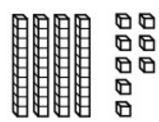
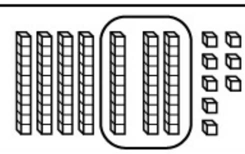
1	2	3	4	●	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	●	24	25	26	27	●	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	●	57	58	59	60
61	62	63	64	65	66	67	68	69	●
●	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	●	95	96	97	98	99	100

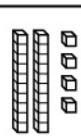
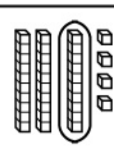
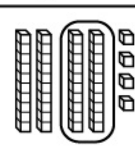
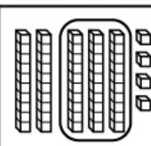
In the second phase, show students a 100s chart where sequences of several numbers are removed from a few different rows or columns, and ask students to identify the missing numbers.

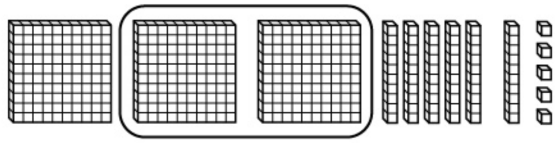
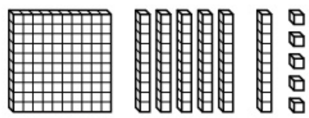
1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	●	●	●	●	●
21	●	23	24	25	26	27	28	29	30
31	●	33	34	35	36	37	38	39	40
41	●	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	●	59	60
61	62	63	64	65	66	67	●	69	70
71	72	73	74	75	76	77	●	79	80
81	●	●	●	●	86	87	●	89	90
91	92	93	94	95	96	97	98	99	100

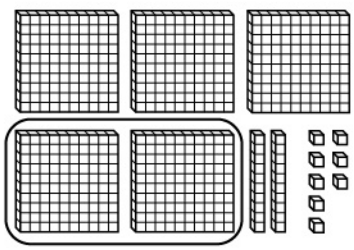
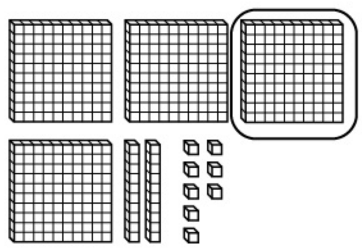
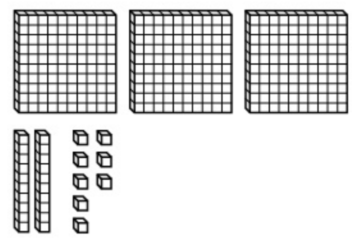
In the next phase, show students a 100s chart where the numbers in all but one or two rows or columns have been removed. Finally, extend this activity to a 1000s chart using the same phases with three-digit numbers. During all phases of this activity, ask students to describe how the missing numbers are found and explain any numerical patterns found. For example, the number directly below 13 is 10 more (23), because of the structure of the 100s chart. Each row has ten numbers.

- Ask students to find a skip-counting rule for changing a digit of a given number. The students are given two numbers in which only one digit is different. For example, 48 and 78, or 365 and 165. Use manipulatives such as base-ten blocks, ten-frame cards, number lines, and/or 100s and 1000s charts to help find a skip-counting rule to go from the first number to the second number. Draw and record the process used to find the skip-counting rule. Next, ask students to model and record two more numbers with the same skip-counting rule. Then, discuss the patterns noticed when changing digits in each place and when finding different numbers with the same skip-counting rule.

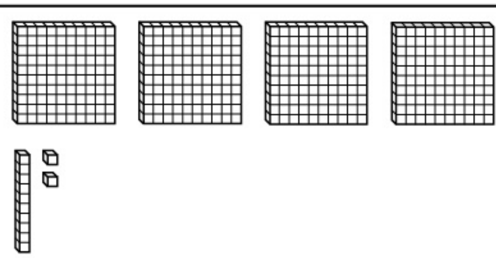
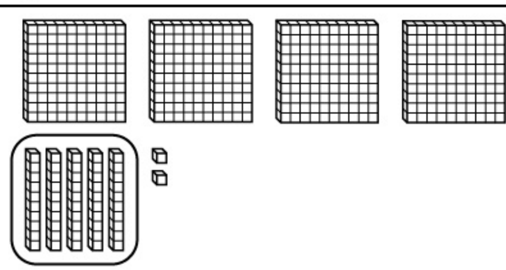
Starting Number	Representation	Skip-Count Rule
48		48, 58, 68, 78
Ending Number 78		Skip-count by 10 three times to go from 48 to 78.

The same rule works to go from 24 to 54.			
 24	 34	 44	 54

Starting Number	Representation	Skip-Count Rule
365		365, 265, 165
Ending Number 165		Skip-count backwards by 100 two times to go from 365 to 165.

The same rule works to go from 528 to 328.		
 528	 428	 328

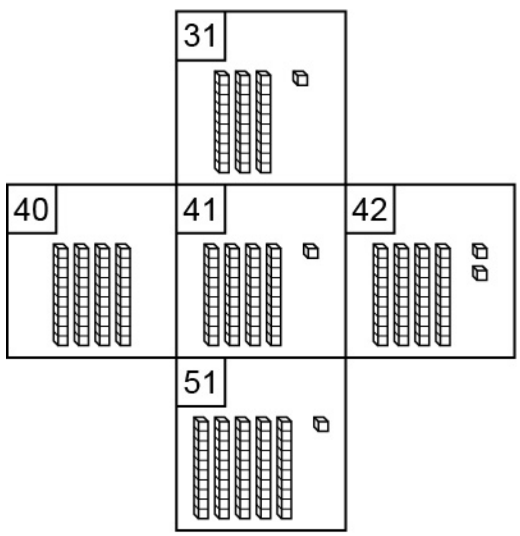
Students who need additional support may be given a set of counting rule cards that say "count by 1," "skip-count by 10," "skip-count by 100," "count backward by 1," "skip-count backward by 10" and "skip-count backward by 100." Each set of "rule" cards should contain at least five cards of each statement. Use the "rule" cards, along with number cards, to build the rule to show how to get from the first number to the second number.

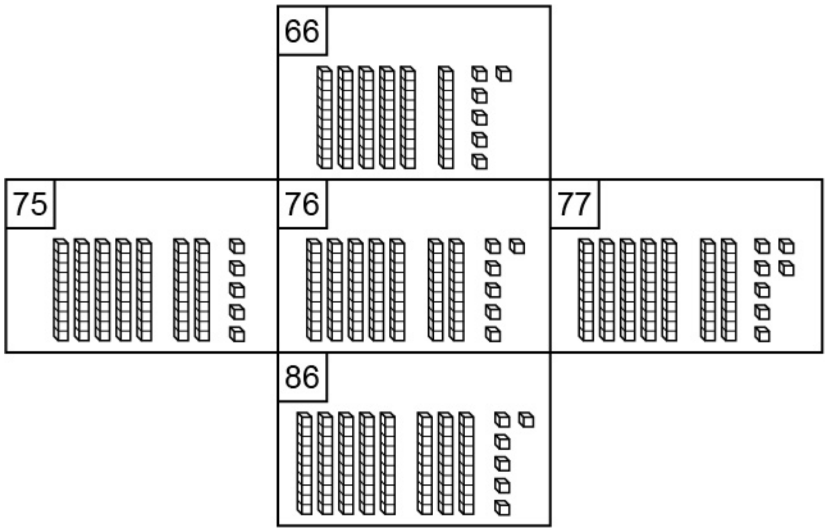
Starting Number	Representation	Skip-Count Rule
412		412 | skip-count by 10 | 422 422 | skip-count by 10 | 432 432 | skip-count by 10 | 442 442 | skip-count by 10 | 452
Ending Number 452		Skip-count by 10 four times to go from 412 to 452.

How are skip-counting by 5s, 10s, and 100s all related?

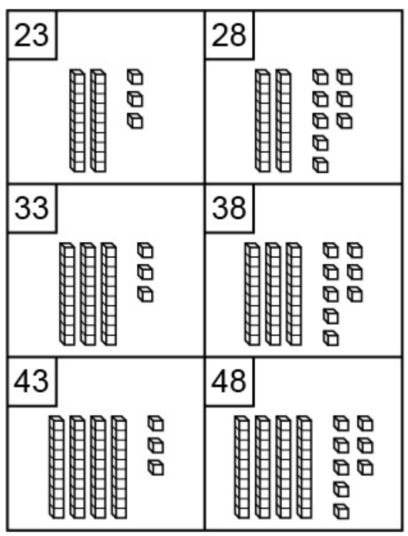
M.P.7. Look for and make use of structure. Find the pattern for skip-counting by 10s within the pattern of skip-counting by 5s by looking at every other number, and find the pattern for skip-counting by 100s within the pattern of skip-counting by 10s by looking at every tenth number. For example, this list skip-counts by 5s starting at 6: <u>6</u>, 11, <u>16</u>, 21, <u>26</u>, 31. The underlined numbers are skip-counting by 10s, starting at 6. The patterns become apparent when using a 100s chart. Additionally, skip-counting by 5s two times and skip-counting by 10 one time from the same number yields the same result. Further, skip-counting by 10s ten times and skip-counting by 100 one time from the same number yields the same result.

- Ask students to create models of numbers, locate the numbers on a 100s chart, and change the models to make a number neighbor. The students are given base-ten manipulatives, such as base-ten blocks or ten-frame cards. Model two or three given numbers in either the same row or same column of the 100s chart. Next, use the manipulatives to show what needs to be changed to make each number neighbor on the 100s chart (the numbers directly to the left, right, above and below the original number). Then, draw the models and record the solutions. Ask students to discuss the solution process and any patterns noticed.



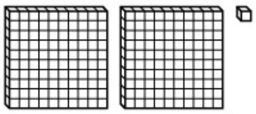
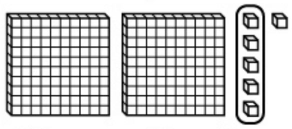
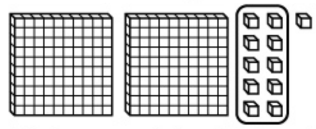
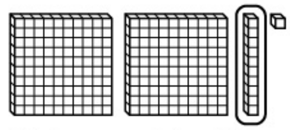


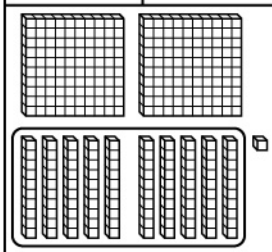
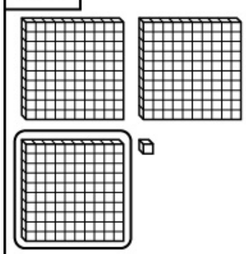
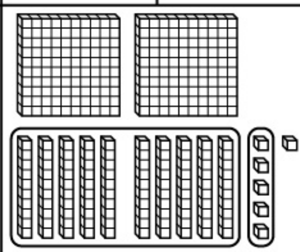
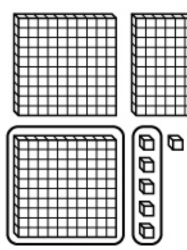
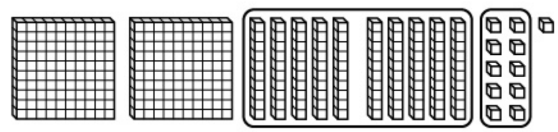
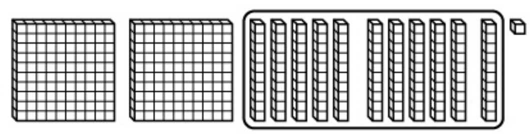
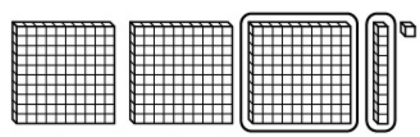
Next, repeat this activity with numbers that have a difference of 5, such as 23, 28, 33, 38, 43, and 48. Ask students to discuss how the models for each number are related and what changes occur between the models.



Extend this activity to the 1000s chart and repeat with groups of two- and three-digit numbers that have differences of 5, 10, and 100.

- Ask students to create models of numbers to show the relationships between skip-counting by 5s two times and skip-counting by 10 one time and between skip-counting by 10s ten times and skip-counting by 100 one time. The students are given base-ten blocks to represent a starting number. Use the base-ten blocks to represent a number that is 5 more than the starting number and a number that is 10 more than the starting number. Model the regrouping of two groups of 5 into one group of 10, drawing and recording the process. Next, use the base-ten blocks to represent a number that is 100 more than the starting number. Then, model the regrouping of ten groups of 10 into one group of 100, drawing and recording the process. The activity can be extended into representing a number that is 105 more than the start number and then a number that is 110 more than the start number. Model the regrouping of the groups of 5 and the groups of 10, drawing and recording the process. Ask students to share and discuss the process and patterns noticed. (Example is on the next page.)

Start	Start	End	Start	End
201	201	206	201	211
	 Skip-count by 5 one time.		 Skip-count by 5 two times. OR  Skip-count by 10 one time.	

Start	End	Start	End	Start	End
201	301	201	306	201	311
 Skip-count by 10 ten times. OR  Skip-count by 100 one time.		 Skip-count by 10 ten times and skip-count by 5 one time.  Skip-count by 100 one time and skip-count by 5 one time.		 Skip-count by 10 ten times and skip-count by 5 two times. OR  Skip-count by 10 eleven times. OR  Skip-count by 100 one time and skip-count by 10 one time.	

Key Academic Terms:

skip-count, place value, numerical pattern, digit, single-digit number, double-digit number, three-digit number

Additional Resources:

- Video: Skip-counting
- Video: Skip-counting by 10 song
- Activity: Count by fives
- Lesson: Skip-count by 10s and 100s
- Article: 3 super tips for teaching place value
- Activity: Choral counting

8

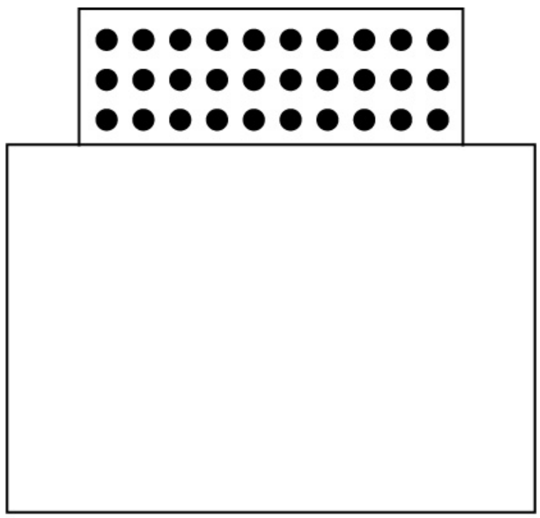
Operations with Numbers: Base Ten
Understand place value.
8. Read and write numbers to 1000 using base-ten numerals, number names, and expanded form.

Guiding Questions with Connections to Mathematical Practices:

How can place value understanding help to represent numbers using number names?

M.P.7. Look for and make use of structure. Connect place value with the names of each place represented in a number to know how to say the number. For example, the number 357 has a 3 in the hundreds place, a 5 in the tens place, and a 7 in the ones place, so it can be read as "three hundred fifty-seven." Additionally, connect base-ten concepts with standard language and verbal number names. For example, 849 is the same as eight hundreds, four tens, and nine ones and is also the same as "eight hundred forty-nine."

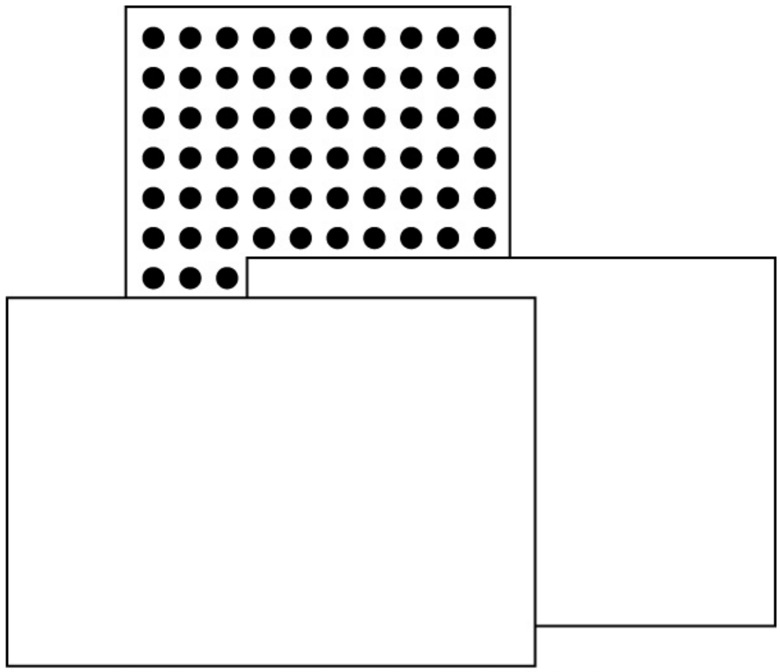
- Ask students to count rows of tens using a 10 by 10 array of dots. Show a 10 by 10 array of dots where all but two rows of dots are covered. This can be done on a projector with a piece of paper used to cover the rows. Ask students how many tens are shown (two). Say “two tens is called twenty,” and ask students to repeat the sentence. Uncover another row of dots and ask students how many tens are shown (three). Say “three tens is called thirty.” Continue this process for forty and fifty. Then, before showing six rows of dots, ask “how many tens does sixty have?” Share any patterns noticed at this time. Continue with seventy, eighty, and ninety. Slide the piece of paper up and down, exposing different numbers of rows of ten and ask students to name the quantity of dots. The paper should also be turned 90 degrees to show columns of tens. Ask students to discuss the patterns noticed.



Three tens is called thirty.

Continue the activity by working on number names for tens and ones. For example, show six full rows of dots and say “six tens is called sixty.” Then, show one more dot in the seventh row. Say “six tens and one is called sixty-one.” Uncover an additional dot and say “six tens and two is called sixty-two.” Continue to uncover dots while saying the base-ten and number names out loud. Discuss the patterns.

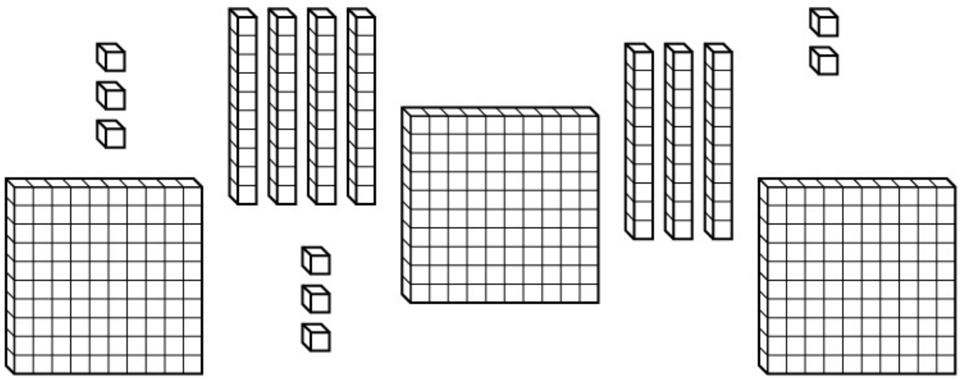
Continue to show different numbers of rows and single dots while students name the numbers out loud. Spend time with students differentiating numbers that may sound similar, such as “fifteen” and “fifty.”



Six tens and three is called sixty-three.

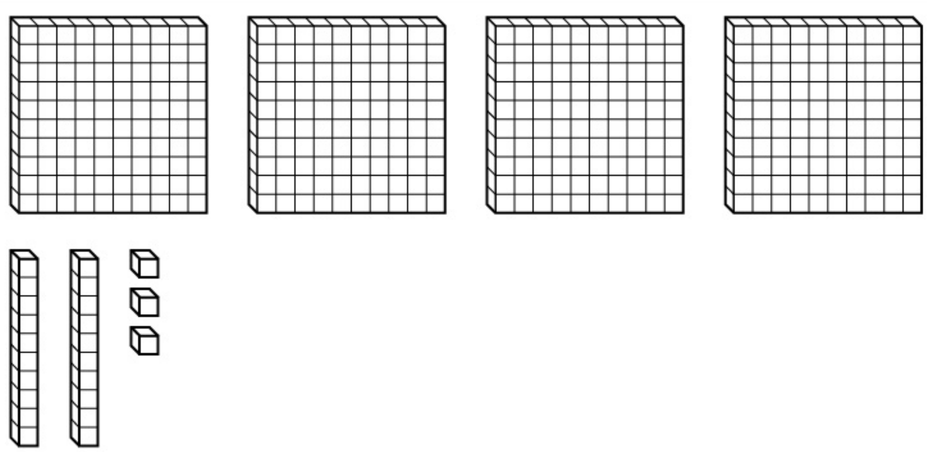
Extend this activity to three-digit numbers using small 10 by 10 arrays to represent groups of hundreds.

- Ask students to count mixed arrangements of base-ten manipulatives, such as base-ten blocks, connector cubes, or groupable counters (with some pre-grouped in cups or bags of 10 and 100). Start with groups of hundreds. Ask students to give the base-ten name and the standard number name. Add or remove a hundred and give the new base-ten name and the standard number name. Next, add some tens and ask for the new base-ten name and standard number name. Then, add some ones. Continue changing the number by changing only the hundreds or only the tens or only the ones. Avoid the standard left–to-right order so the emphasis is on the names of the groups rather than the order of the manipulatives.



The number shown is three hundreds, seven tens, and eight ones.
The number shown is three hundred seventy-eight.

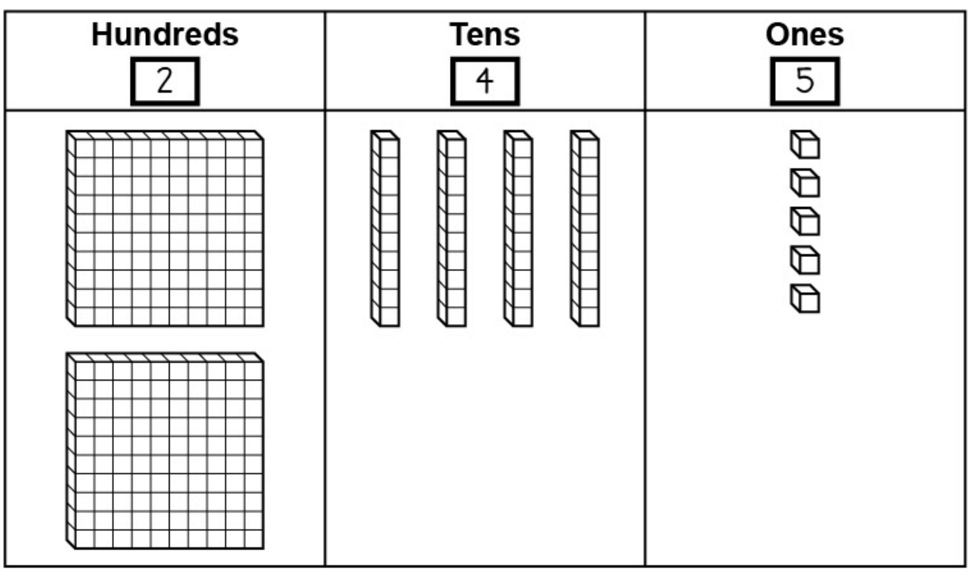
Reverse the activity by having students use the manipulatives to build a number given to them verbally. The students build the number, draw a representation, and record the number using base-ten language and the standard number name. Include examples with zero in the tens and ones places.

Number Name
four hundred twenty-three

4 hundreds, 2 tens, 3 ones = 423

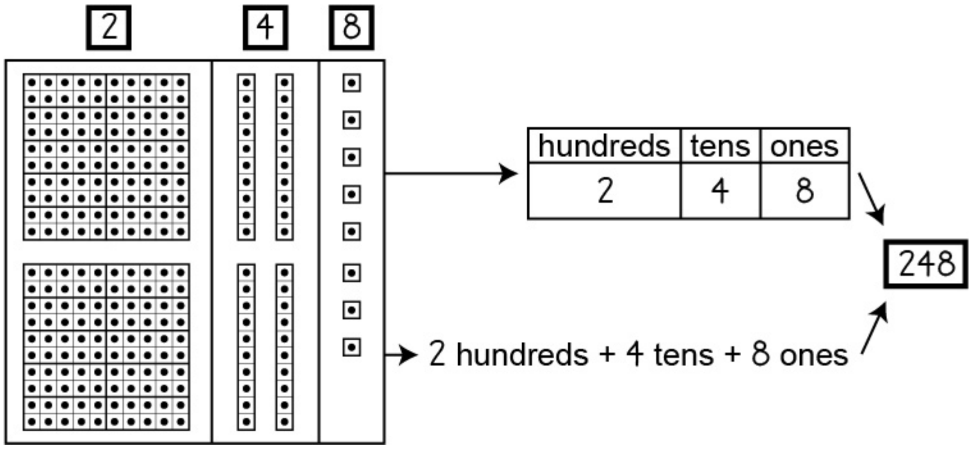
How can place value understanding help to write numerals from numbers represented in standard (number name) form?

M.P.7. Look for and make use of structure. Identify that number names relate to place value and can be used to determine the location of digits. For example, seven hundred twenty-nine has a 7 in the hundreds place, a 2 in the tens place, and a 9 in the ones place. As such, the number seven hundred twenty-nine can be written as 729. Additionally, the symbolic system used for writing numbers (ones on the right, tens to the left of ones, hundreds to the left of tens, etc.) can be coordinated with grouping methods. Further, the digit represents the number of groups for each place.

- Ask students to use place value mats and manipulatives, such as base-ten blocks, bundles of straws, or mini ten-frames, to make groups to match given numerals for each place. Place value mats can be made on sheets of paper, individual white boards, or the floor using masking tape. For this activity, the place value mats should be divided into three sections to hold ones, tens, and hundreds. The sections should be large enough for students to place manipulatives on as they build a number. For students who need additional support, ten-frames can be placed or drawn in the ones place to make the quantity of ones clear and to avoid the need to repeatedly recount them. This also helps reinforce the concept of groups of ten. Two ten-frames may be used if the students are building more than one number at a time.



The students are given a numeral to assign to each place on their place value mats. Next, use manipulatives to build the number of groups for each place. Then, draw and record the numerals in labeled places and write the number in standard form.



- Ask students to count a large number of objects (between 50 and 999 objects). Each group of students is given a bag of objects such as toothpicks, beads, buttons, or counters. Record an estimate of the number of objects in the bag using base-ten names; for example, 3 hundreds, 4 tens, and 6 ones for a guess of 346 objects. Then, group the objects and record the actual number of groups for each place, along with the standard form of the number. The number name can also be recorded in order to include all place value components.

Example Recording Sheet

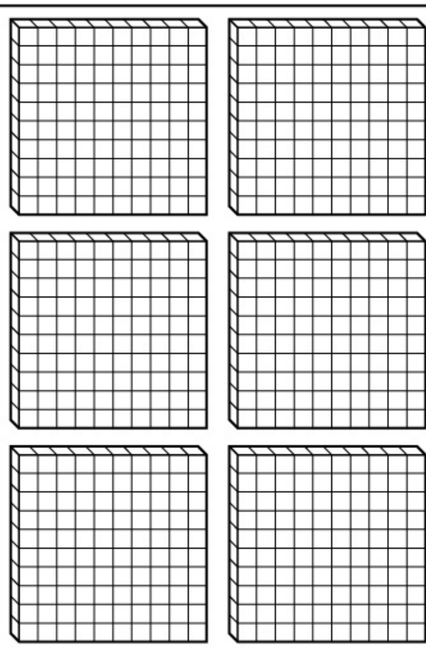
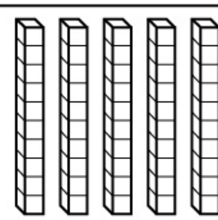
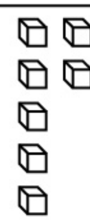
Object	Estimate	Actual
buttons	5 hundreds, 3 tens, 6 ones	2 hundreds, 8 tens, 3 ones standard form 283 number name two hundred eighty-three
toothpicks	7 hundreds, 8 tens, 3 ones	5 hundreds, 9 tens, 7 ones standard form 597 number name five hundred ninety-seven

When students finish counting the objects in the bag, the bags of objects can be traded among groups.

How can a number be represented in expanded form?

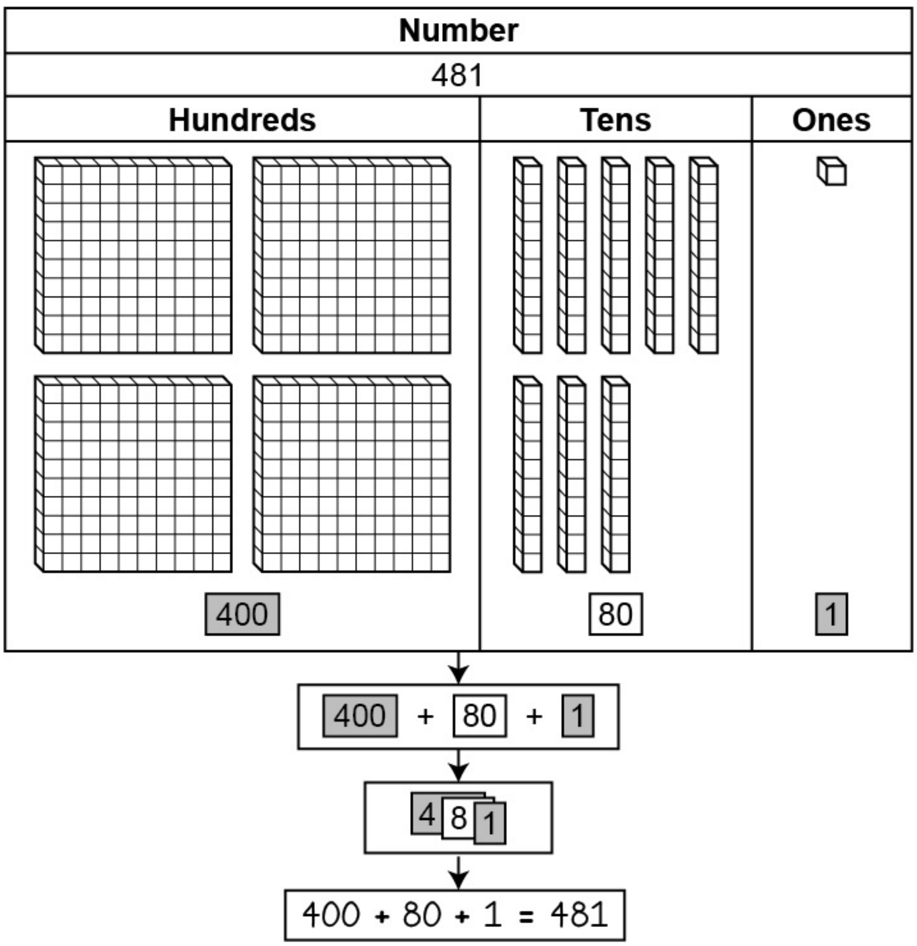
M.P.7. Look for and make use of structure. Decompose a number into digits and attach value to each digit to represent the number in expanded form. For example, 408 has the digit 4 in the hundreds place (i.e., 400), the digit 0 in the tens place, and the digit 8 in the ones place; therefore, the expanded form of 408 is 400 + 8. Additionally, identify that the positions of digits in numbers determine what value the digits represent.

- Ask students to use place value mats and manipulatives such as base-ten blocks or groupable counters to build given three-digit numbers. Draw the representation and record the quantity for each place. Then, use the quantities for each place to write the number in expanded form.

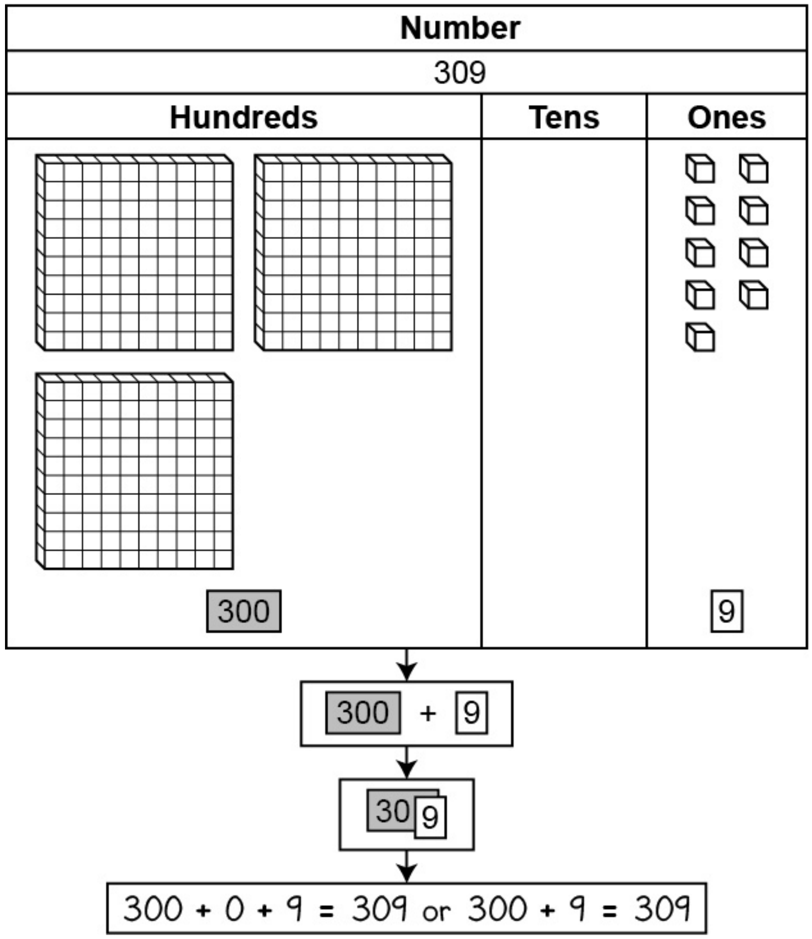
Number		
657		
Hundreds	**Tens**	**Ones**
		
6 hundreds = 600	5 tens = 50	7 ones = 7
Expanded form 600 + 50 + 7 = 657		

Include examples with zero in the tens place, such as 803, to help students confirm the importance of zero in place value. Discuss how the zero helps distinguish between numbers such as 803, 83, and 830.

- Ask students to build numbers using base-ten manipulatives and sets of number cards. The students are given a set of cards that include one card for each hundred (100–900), one card for each ten (10–90), and ones cards (1–9). The students then represent three-digit numbers using base-ten manipulatives, such as base-ten blocks. As students position the base-ten blocks for the number (for example, 481), the students also place the matching number cards (400, 80, and 1) below the corresponding base-ten blocks in order to connect the model to the numerical representation. Next, use the number cards to write the number in expanded form. The students can then build the number starting with the hundreds card and placing the tens and ones cards on top, keeping the cards right-aligned so each digit for each place is visible. After the students compose the number, the cards can be taken apart to model the decomposition of the number.



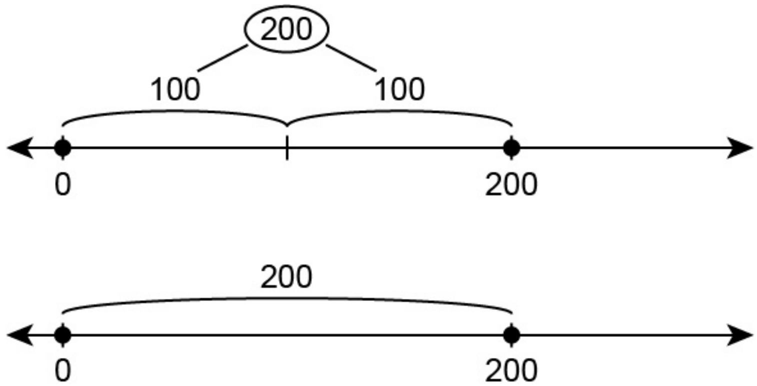
Include examples of numbers with zero in the tens place. Discuss the role of zero in place value.



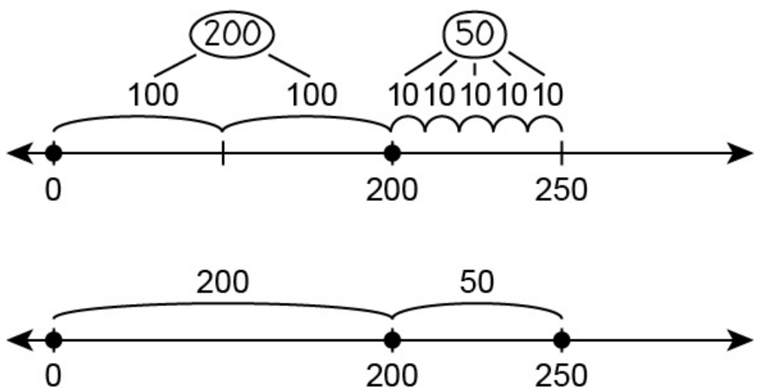
How can a visual representation connect the different forms of a number?

M.P.4. Model with mathematics. Use drawings and manipulatives to represent a number in different forms. For example, the number 245 can be represented by 2 hundreds ("flats"), 4 tens ("rods"), and 5 ones ("units"), which can be written as 2 hundreds + 4 tens + 5 ones. Additionally, understand that a number represented in different forms has the same value regardless of the representation used.

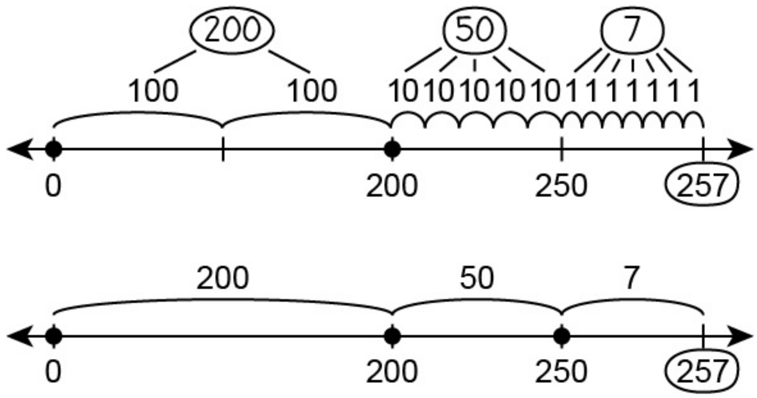
- Ask students to represent a three-digit number on a number line using different place value components of the number. The students are given a three-digit number to represent on a number line, for example 257. The students begin at zero on the number line. Use the value of the digit in the hundreds place to model a “jump” (or skip-count jumps) to that value on the number line. Plot a point and label the value. For example, the number 200 is shown.



Next, use the value of the digit in the tens place to model a “jump” that many more from the hundreds value. The students plot a point and label the value. For example, the number 250 is shown.

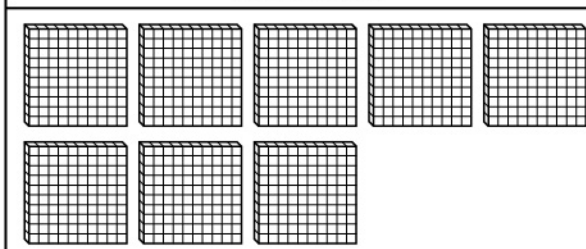
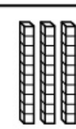
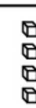


The students then model a “jump” for the value of the ones and plot a point and label the value. For example, the number 257 is shown, by starting with the original given number, 200.



The modeling on the number line may not be to scale, but students should demonstrate that a hundreds “jump” is bigger than a tens “jump” and a tens “jump” is bigger than a ones “jump.”

- Ask students to represent numbers using manipulatives, drawings, base-ten language, the values of the digits, expanded form, standard form, and number names. The students are given a three-digit number to represent using base-ten manipulatives. Then, draw the model and record the number of groups and the value for each place. Next, use the recorded values for each place to write the expanded form of the number. Then, record the standard form of the number, along with the number name. Discuss the similarities and differences between the representations, and the changes needed in order to move from one representation to another.

	Hundreds	**Tens**	**Ones**
Model			
Base-Ten	8 hundreds	3 tens	4 ones
Place Value	800	30	4
Expanded Form	800 + 30 + 4		
Standard Form	834		
Number Name	eight hundred thirty-four		

This activity can be extended by giving students pieces of information for some of the forms. Then, use manipulatives, place value knowledge, and things discussed previously to fill in the missing information. The students can also create their own charts with pieces of information to give to another student to fill in.

	Hundreds	Tens	Ones
Model			
Base-Ten	6 hundreds	__ tens	__ ones
Place Value		50	
Expanded Form			
Standard Form			
Number Name			

Key Academic Terms:

place value, base-ten numeral, number name, expanded form, standard form, digit, decompose

Additional Resources:

- Lesson: [Place value concepts]
- Worksheets: [Place value for 2nd grade]

9

Operations with Numbers: Base Ten
Understand place value.
9. Compare two three-digit numbers based on the value of the hundreds, tens, and ones digits, recording the results of comparisons with the symbols >, =, and < and orally with the words “is greater than,” “is equal to,” and “is less than.”

Guiding Questions with Connections to Mathematical Practices:

How can place value understanding help to compare three-digit numbers?

M.P.7. Look for and make use of structure. Compare place value between two numbers to decide which number is greater and then represent the comparison using the appropriate symbol and vocabulary. For example, 635 is greater than 580 because 635 has 6 hundreds and 580 has 5 hundreds. Similarly, even though 635 and 642 have the same number of hundreds, 635 is less than 642 because the former only has 3 tens and the latter has 4 tens. These comparisons can be represented with symbols as $635 > 580$ and $635 < 642$. Additionally, an equal sign can be used to show that two numbers have the same value.

- Ask students to compare two three-digit numbers that differ in the hundreds place. For example, 587 is greater than 499 because 587 has 5 hundreds and 499 has 4 hundreds. Note that the number of tens and ones that each of these three-digit number has does not affect which of them is greater because the number of hundreds is different.

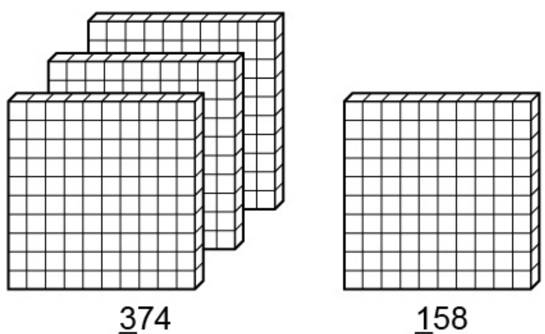
- Ask students to compare two three-digit numbers with the same number of hundreds but that differ in either the tens or ones place (or both). For example, a comparison of 865 and 819 should begin by comparing the digits in the hundreds place, but since both numbers have 8 hundreds, the comparison should move to the next place to the right—the tens place. Because 865 has 6 tens and 819 only has 1 ten, 865 is greater than 819. Again, note that the number of ones does not affect which of the numbers is greater because the number of tens is different.

- Ask students to record the results of their comparisons using $>$, $=$, and $<$. Examples include:
    - $509 < 621$ because 621 has 6 hundreds and 509 has 5 hundreds.
    - $185 > 183$ because while both numbers have 1 hundred and 8 tens, 185 has 5 ones and 183 has 3 ones.
    - $327 = 327$ because both numbers have 3 hundreds, 2 tens, and 7 ones.

How can visual representations help compare numbers?

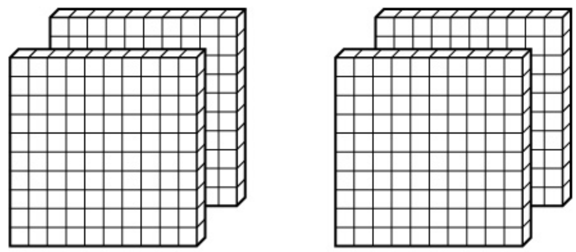
M.P.4. Model with mathematics. Use drawings or manipulatives to help compare numbers. For example, when comparing 289 and 245, draw two hundreds ("flats") first. Since they are the same, move to the next largest place value and draw 8 tens ("rods") and 4 tens ("rods"). Since there are more tens in 289 than 245, $289 > 245$. Additionally, demonstrate that it is not necessary to continue modeling a number past the point where the place value that is being compared has different digits.

- Ask students to compare 2 three-digit numbers with different numbers of hundreds using manipulatives. For example, when comparing 374 and 158, first lay out enough hundreds blocks to represent the number of hundreds present in each number, as shown.



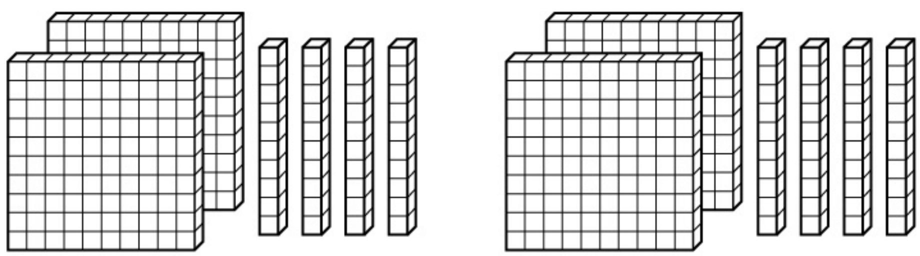
In this case, because the number of hundreds blocks needed is different for the two numbers being compared, no more blocks need to be used. Ask students to write the correct comparison using the appropriate symbol, such as $374 > 158$.

- Ask students to use manipulatives to compare two three-digit numbers with different numbers of tens or ones but the same number of hundreds. For example, when comparing 245 to 241, first lay out and compare enough hundreds blocks to represent the number of hundreds present in each number, as shown.



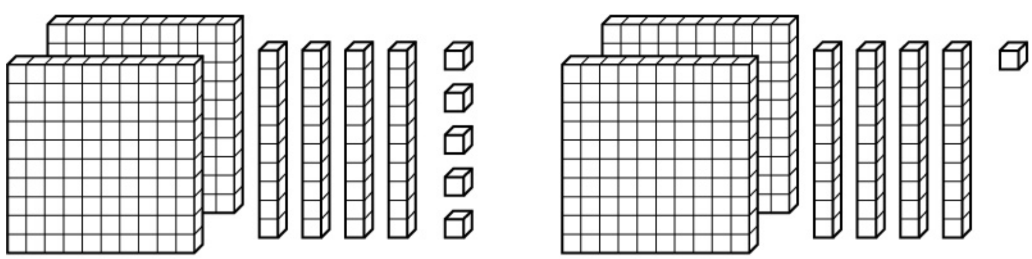
245 241

Then, since the number of hundreds blocks for each number are equal, lay out and compare enough tens to represent the number of tens present in each number, as shown.



245 241

Finally, since the number of tens blocks for each number are equal, lay out and compare enough ones to represent the number of ones present in each number, as shown.



245 241

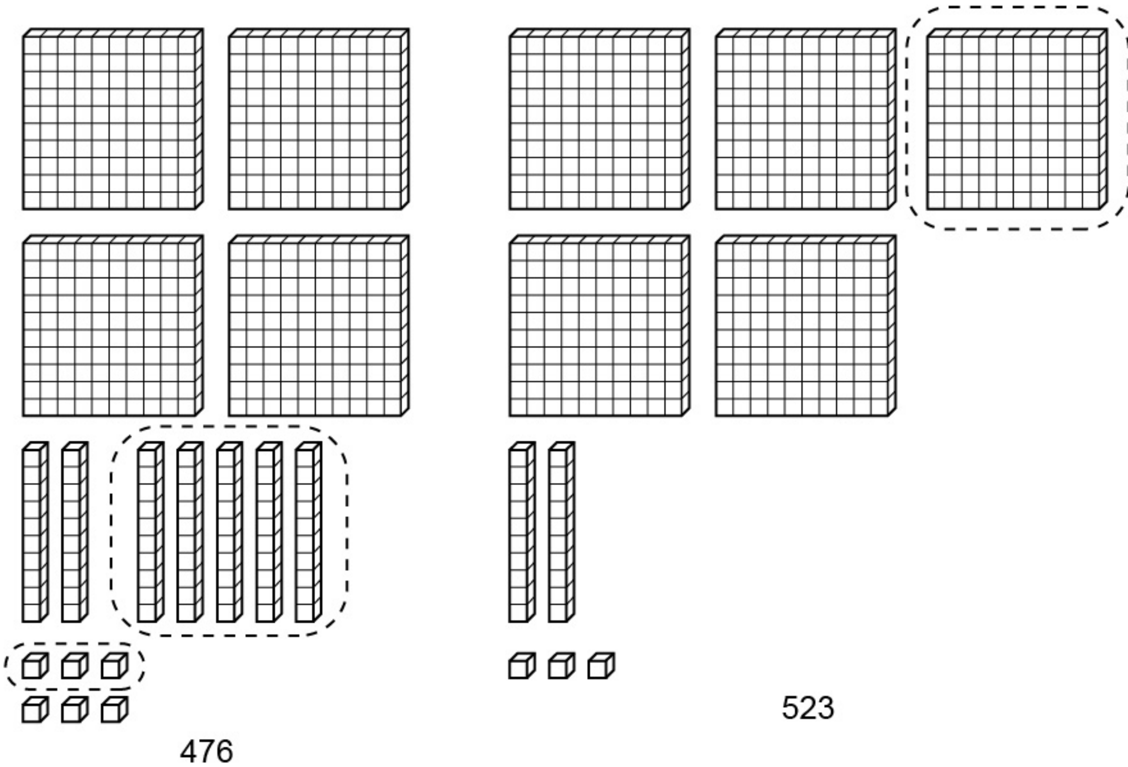
Because the number of ones needed to represent 245 is greater than the number of ones needed to represent 241 (and the numbers of hundreds and tens for each number were equal), 245 is greater than 241. Ask students to write the correct comparison using the appropriate symbol, such as $245 > 241$.

Why is the place value farthest to the left compared first when comparing three-digit numbers?

M.P.3. Construct viable arguments and critique the reasoning of others. Explain why it's important to compare the farthest left place values first when comparing numbers. For example, explain that the value of the digit in the hundreds place is greater than any single digit number of tens or ones; therefore, when comparing two 3-digit numbers, if the value in the hundreds place is greater in one number, then there is no need to compare the values in the remaining places. Additionally, demonstrate that, given two numbers, the lesser of the two can have greater values in the tens and ones places.

- Ask students to explain what it means for a number to be greater than another number. For example, why is 24 greater than 17? When represented using base ten blocks, 24 has more unit cubes in it than 17 does, regardless of how the individual digits in the number compare.

- Ask students to explain why it is not necessary to compare tens digits or ones digits when the hundreds digits are not equal. For example, ask students to compare 476 and 523 and discuss why there is no need to compare the tens and ones digits. A visual representation is shown.



Observe that 523 has an additional hundreds block that 476 lacks and that 476 has additional tens and ones blocks that 523 lacks. The extra hundreds block that 523 has adds a value of 100 units, but the extra tens and ones blocks that 476 has only add a value of 53. Therefore, 523 is larger despite the additional blocks that 476 has. Further, because the digits 0 to 9 are the only digits used when writing numbers, the most extra value a number can gain from extra tens and ones blocks is $90 + 9 = 99$, which can never outweigh an extra hundreds block.

Key Academic Terms:

place value, digit, compare, greater than (>), equal to (=), less than (<), three-digit number

Additional Resources:

- Activity: Place value challenge
- Worksheets: Place value for 2nd grade

10

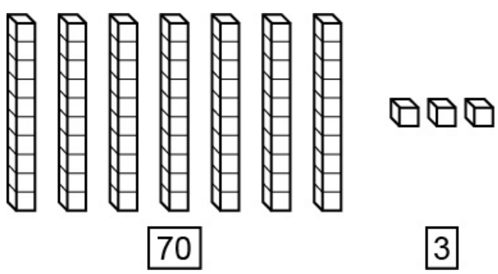
Operations with Numbers: Base Ten
Use place value understanding and properties of operations to add and subtract.
10. Fluently add and subtract within 100, using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

Guiding Questions with Connections to Mathematical Practices:

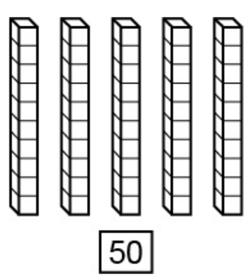
How can place value understanding help to fluently add and subtract numbers?

M.P.7. Look for and make use of structure. Decompose numbers into tens and ones and add each place value grouping together, then add the sums together. For example, $46 + 25$ is the same as adding 6 ones and 5 ones for a total of 11 ones, and then adding 4 tens and 2 tens for a total of 6 tens. Adding 11 ones and 6 tens gives a sum of 71. Additionally, quantities of 10 or more ones can be regrouped into a single ten and a number of ones.

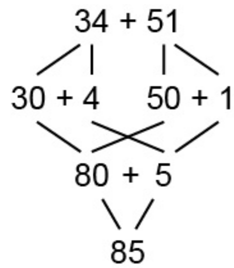
- Ask students to identify the number of tens and ones in a given number and represent them using base-ten blocks. For example, students should verify that the number 73 is comprised of 7 tens and 3 ones. This decomposition is represented by the base-ten blocks shown.



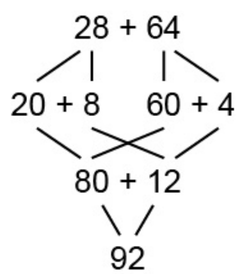
Further, verify that the number 50 is comprised of 5 tens and 0 ones. This decomposition is represented by the base-ten blocks shown.



- Ask students to decompose numbers in an addition problem into tens and ones and add each place value separately in situations without regrouping. For example, in the sum $34 + 51$, the number 34 can be decomposed into 3 tens and 4 ones while the number 51 can be decomposed into 5 tens and 1 one. The number of ones in the sum is 5, since 4 ones combined with 1 one gives 5 ones. The number of tens in the sum is 8, since 3 tens combined with 5 tens gives 8 tens. The sum has 8 tens and 5 ones, therefore $34 + 51 = 85$.

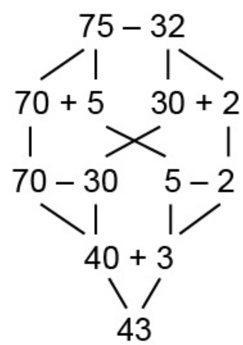


- Ask students to decompose numbers in an addition problem into tens and ones and add each place value separately in situations with regrouping. For example, in the sum $28 + 64$, the number 28 can be decomposed into 2 tens and 8 ones while the number 64 can be decomposed into 6 tens and 4 ones. Combining 8 ones and 4 ones gives 12 ones. The 12 ones can be regrouped as 1 ten and 2 ones. Now, the number of ones in the sum is 2, while the number of tens in the sum is 9 (2 tens from 28, 6 tens from 64, and 1 ten from the regrouping of 12). Therefore, the sum has 9 tens and 2 ones, so $28 + 64 = 92$.

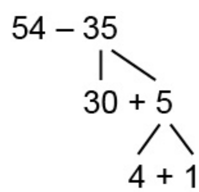


M.P.7. Look for and make use of structure. Decompose the minuend or subtrahend in a subtraction problem to use place value for subtraction. For example, 61 – 18 can be thought of as 61 – 10 (i.e., 51), and then subtract 1 from the answer of 51 to get to 50. Then, use the remaining 7 ones to get the final difference of $50 - 7 = 43$. Additionally, a ten can be regrouped to make 10 ones when necessary.

- Ask students to decompose parts of a subtraction problem and subtract each place value separately in situations where the number of ones in the subtrahend is less than that of the minuend. For example, ask students to subtract 75 – 32. The number 75 can be decomposed into 7 tens and 5 ones. The number 32 can be decomposed into 3 tens and 2 ones. Then, the 2 ones can be subtracted from the 5 ones, leaving 3 ones. Likewise, the 3 tens can be subtracted from the 7 tens, leaving 4 tens. The difference has 4 tens and 3 ones, so 75 – 32 = 43.



- Ask students to decompose the subtrahend and subtract the subtrahend over multiple steps in situations where the number of ones in the subtrahend is greater than that of the minuend. For example, ask students to subtract 54 – 35. The subtrahend of 35 can be decomposed into 3 tens and 5 ones. The 5 can be further decomposed to get 30 + 4 + 1 (where the 4 is chosen to match the ones digit of the minuend). The difference can be calculated starting with 54 – 30 to get 24, and then 24 – 4 to get 20, and finally 20 – 1 to get 19. Therefore, 54 – 35 = 19.



54 – 30 = 24
24 – 4 = 20
20 – 1 = 19

- Ask students to exchange a ten for 10 ones when performing subtraction. For example, ask students to subtract 63 – 49. The number 63 can be decomposed into 6 tens and 3 ones. The number 49 can be decomposed into 4 tens and 9 ones. Because 9 ones is greater than 3 ones, exchange 1 of the tens in 63 for 10 ones, so 63 is now composed of 5 tens and 13 ones. Then, the 9 ones can be subtracted from the 13 ones, leaving 4 ones. Likewise, the 4 tens can be subtracted from the 5 tens, leaving 1 ten. The difference has 1 ten and 4 ones, so 63 – 49 = 14.

How can the properties of operations help to fluently add and subtract numbers?

M.P.2. Reason abstractly and quantitatively. Decompose numbers and use the properties of operations to add or subtract the parts of the numbers. For example, 62 + 29 can be expanded to 60 + 2 + 20 + 9 and then rearranged to 60 + 20 + 2 + 9 = 80 + 11 = 91. Additionally, decompose ones into groups of numbers that combine to make 10 to add numbers.

- Ask students to decompose numbers to add 2 two-digit numbers. For example, 53 + 46 can be decomposed into 50 + 3 + 40 + 6. This expression can be rearranged to 50 + 40 + 3 + 6. Once rearranged, the addition of tens and ones is easier to see. Adding the tens and ones separately gives 90 + 9, so 53 + 46 = 99. Another possible method is shown.

53 + 46

50 + 40 = 90

90 + 3 = 93

93 + 6 = 99

- Ask students to decompose numbers to subtract 2 two-digit numbers. For example, 79 – 46 can be decomposed into 70 + 9 – 40 – 6. This expression can be rearranged to 70 – 40 + 9 – 6, taking care to keep the subtraction signs associated with the correct values. Once rearranged, the subtraction of tens and ones is easier to see. Subtracting the tens and ones separately gives 30 + 3, so 79 – 46 = 33.

79 – 46

70 – 40 = 30

9 – 6 = 3

30 + 3 = 33

- Ask students to decompose the subtrahend of a subtraction problem. For example, give students the problem 65 – 37. The 37 can be decomposed as 30 + 7, which allows the problem to be rewritten as 65 – 30 – 7. The first subtraction reduces the problem to 35 – 7, and the second subtraction gives the solution of 28.

$$65 - 37$$

$$65 - 30 = 35$$

$$35 - 7 = 28$$

How does the relationship between addition and subtraction help to fluently subtract numbers?

M.P.2. Reason abstractly and quantitatively. Rewrite a subtraction problem as a missing addend problem. For example, 49 – 21 can be rewritten as 21 + ☐ = 49. Confirm that 21 + 20 = 41 and 41 + 8 = 49. Add the 20 + 8 to get a sum of 28. Because 21 + 28 = 49, know that 49 – 21 = 28. Additionally, demonstrate that regrouping when adding ones may be necessary when solving an addition problem with a missing addend.

- Ask students to rewrite subtraction equations as addition equations. For example, the subtraction equation 86 – 35 = 51 can be rewritten as 51 + 35 = 86. It can also be rewritten as 35 + 51 = 86. Students should verify that the addends can be written in either order and that the addends in the addition equation should always be the subtrahend and the difference from the subtraction equation.

- Ask students to rewrite a subtraction problem as a missing addend problem and use the addition problem to solve the subtraction problem. For example, 74 – 51 can be rewritten as 51 + ☐ = 74. Then, 51 + 20 = 71 and that another 3 needs to be added to get from 71 to 74. Therefore, 51 + 23 = 74, so 74 – 51 = 23. Likewise, 91 – 26 can be rewritten as 26 + ☐ = 91. In order to end up with 1 one in the sum, 5 ones need to be added to 26. Then, since 26 + 5 = 31, a total of 6 tens need to be added to 26 since 31 + 60 = 91. Therefore, the missing addend is made up of 6 tens and 5 ones, so 91 – 26 = 65.

Key Academic Terms:

place value, addition, subtraction, sum, difference, addend, minuend, subtrahend, decompose, compose, properties of operations

Additional Resources:

- Activity: Race to 100
- Activity: Ford and Logan add 45 + 36
- Activity: Jamir's Penny Jar

11

Operations with Numbers: Base Ten

Use place value understanding and properties of operations to add and subtract.

11. Use a variety of strategies to add up to four two-digit numbers.

Guiding Questions with Connections to Mathematical Practices:

How can place value understanding help to add up to four two-digit numbers?

M.P.2. Reason abstractly and quantitatively. Connect adding two numbers using place value to adding more than two numbers in succession. For example, $53 + 26 + 34 + 17$ decomposed into tens and ones would be: $50 + 20 + 30 + 10$ and $3 + 6 + 4 + 7$. Then add 110 to 20 to total 130. Additionally, regroup a quantity of ones greater than 10 into an appropriate quantity of tens and regroup a quantity of tens greater than 10 into an appropriate quantity of hundreds.

- Ask students to add more than 2 two-digit numbers in which both the total number of ones and the total number of tens do not exceed 10. For example, ask students to add $41 + 12 + 34$. Each of the two-digit numbers can be decomposed into tens and ones. The result is $40 + 10 + 30 = 80$ and $1 + 2 + 4 = 7$, so the sum $80 + 7$ is equal to 87.

- Ask students to add more than 2 two-digit numbers where some regrouping is required. For example, ask students to add $56 + 38 + 79 + 23$. Decomposing each two-digit number and then adding the ones gives the sum $6 + 8 + 9 + 3 = 26$. Record 6 as the number of ones in the sum and include the 2 tens when adding the tens. Adding the tens for the given sum gives $50 + 30 + 70 + 20 + 20$, where the last 20 is the 2 tens from the sum of the ones digits. This gives a total of 190. Add $190 + 6 = 196$, therefore the sum is $56 + 38 + 79 + 23 = 196$.

How can the properties of operations help to add up to four two-digit numbers?

M.P.7. Look for and make use of structure. Connect adding two numbers using the properties of operations to adding more than two numbers in succession. For example, $49 + 26 + 13 + 51$ can be rearranged as $49 + 51 + 26 + 13$ so that it is easier to mentally add the numbers together using benchmarks of 10 or 100. Since $49 + 51 = 100$, and $26 + 13 = 39$, the sum is 139. Additionally, demonstrate that regrouping or moving small numbers of ones from one addend to another can facilitate addition.

- Ask students to add more than 2 two-digit numbers by first reordering the numbers to facilitate mental math strategies. For example, $72 + 36 + 28$ can be rewritten as $72 + 28 + 36$. Because $72 + 28 = 100$, the sum can be thought of as $100 + 36$, which is equal to 136. Likewise, the sum $53 + 26 + 77 + 21$ can be rewritten as $53 + 26 + 21 + 77$. Adding $53 + 26$ gives a new sum of $79 + 21 + 77$. Because $79 + 21 = 100$, the sum can be thought of as $100 + 77$, which is equal to 177.

- Ask students to add more than 2 two-digit numbers by moving numbers of ones from one addend to another (using compensation strategy). For example, in the sum $63 + 36 + 74 + 21$, moving 1 from the 21 to the 36 gives the equivalent sum $63 + 37 + 74 + 20$. Because $63 + 37 = 100$, the sum can be thought of as $100 + 74 + 20$. Because $74 + 20 = 94$, the sum of the original expression is 194.

Key Academic Terms:

place value, addition, subtraction, sum, difference, addend, minuend, subtrahend, decompose, compose, two-digit number, properties of operations

Additional Resources:

- Video: [Add two-digit numbers (grade 2)]
- Video: [Adding up to 4 two-digit numbers]

12a

Operations with Numbers: Base Ten
Use place value understanding and properties of operations to add and subtract.
12. Add and subtract within 1000 using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method. a. Explain that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.

Guiding Questions with Connections to Mathematical Practices:

How do addition and subtraction within 1000 relate to addition and subtraction within 100?

M.P.2. Reason abstractly and quantitatively. Connect place value understanding and the properties of operations used to add and subtract within 100 to the properties of operations used to add and subtract within 1000. For example, just as two-digit numbers can be decomposed into tens and ones, three-digit numbers can be decomposed into hundreds, tens, and ones. Additionally, 10 ones can be grouped together to make 1 ten, and 10 tens can be grouped together to make 1 hundred.

- Ask students to use decomposition to add 2 three-digit numbers with a sum less than 1,000 that does not require regrouping. For example, given $263 + 714$, students can decompose each of the addends as follows:
  - 263 can be decomposed into 2 hundreds, 6 tens, and 3 ones.
  - 714 can be decomposed into 7 hundreds, 1 ten, and 4 ones.

  Then, the sum can be found by adding the ones, tens, and hundreds separately, as follows:
  - $3 + 4 = 7$
  - $60 + 10 = 70$
  - $200 + 700 = 900$

  Therefore, the sum $263 + 714$ is equal to 977.

- Ask students to use decomposition to add 2 three-digit numbers with a sum that does require regrouping. For example, given $358 + 246$, students can decompose each of the addends as follows:

    - 358 can be decomposed into 3 hundreds, 5 tens, and 8 ones.
    - 246 can be decomposed into 2 hundreds, 4 tens, and 6 ones.

  Then, the sum can be found by adding the ones, tens, and hundreds separately, in that order, as follows:

    - $8 + 6 = 14$, which can be regrouped into 1 ten and 4 ones, so the sum will have a 4 in the ones place.
    - $50 + 40 + 10$ (from the regrouping of 14 ones) $= 100$. The 10 tens can be regrouped to make 1 hundred with no tens remaining, so the sum will have a 0 in the tens place.
    - $300 + 200 + 100$ (from the regrouping of 10 tens) $= 600$, so the sum will have a 6 in the hundreds place.

  Therefore, the sum of $358 + 246$ is 604.

- Ask students to use decomposition to subtract 2 three-digit numbers. For example, given $893 - 241$, students can decompose the minuend and the subtrahend as follows:

    - 893 can be decomposed into 8 hundreds, 9 tens, and 3 ones.
    - 241 can be decomposed into 2 hundreds, 4 tens, and 1 one.

  Then, the sum can be found by subtracting the ones, tens, and hundreds separately, as follows:

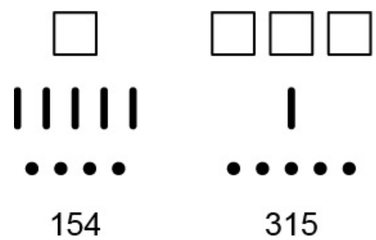
    - $3 - 1 = 2$
    - $90 - 40 = 50$
    - $800 - 200 = 600$

  Therefore, the difference of $893 - 241$ is 652.

How can addition and subtraction within 1000 be modeled in a variety of ways?

M.P.4. Model with mathematics. Model addition and subtraction by using manipulatives and drawings, and use the model to explain thinking. For example, a model to add two three-digit numbers could use drawings of squares to represent hundreds, lines to represent tens, and dots to represent ones. The two numbers are drawn, and then the total number of squares, lines, and dots leads to the number of hundreds, tens, and ones in the sum. Additionally, base-ten blocks can be used to represent hundreds, tens, and ones in the sums and differences.

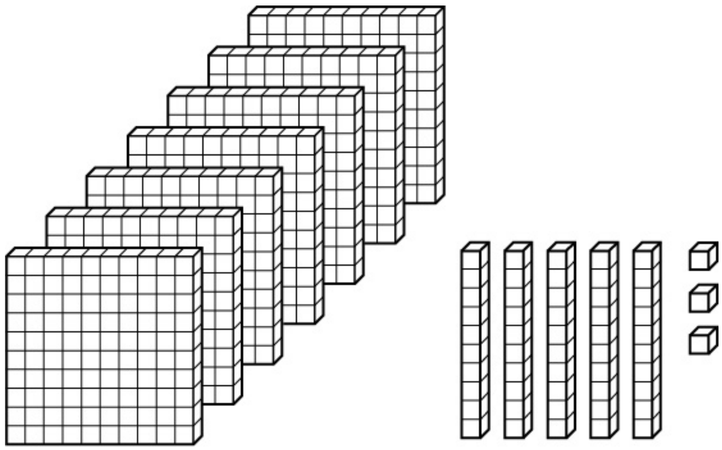
- Ask students to add two three-digit numbers using a visual representation of the numbers. For example, given the sum $154 + 315$, students might use squares, lines, and dots to represent hundreds, tens, and ones, as shown.



  - The total number of dots, $4 + 5$, gives the number of ones in the sum (9).
  - The total number of lines, $5 + 1$, gives the number of tens in the sum (60).
  - The total number of squares, $1 + 3$, gives the number of hundreds in the sum (400).

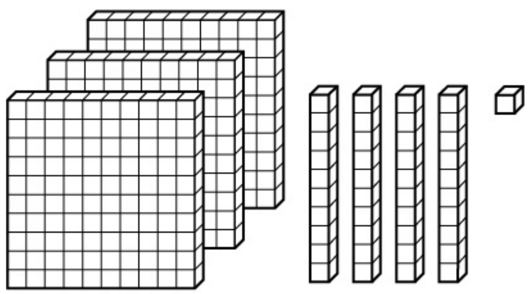
Therefore, the sum $154 + 315$ is equal to 469.

- Ask students to subtract two three-digit numbers using base ten blocks to represent the minuend. For example, given the difference 753 – 412, the minuend 753 can be represented using base ten blocks as shown.



753

The subtrahend 412 can be decomposed into 4 hundreds, 1 ten, and 2 ones. Removing the corresponding base-ten blocks from the previous arrangement leaves the following arrangement.



341

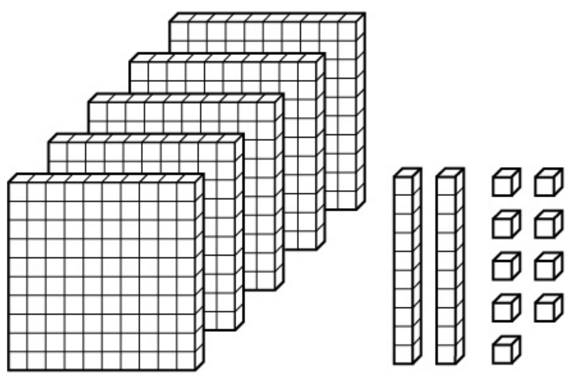
This arrangement shows 3 hundreds, 4 tens, and 1 one, so the difference, 753 – 412, is equal to 341.

When does a hundred or ten need to be decomposed in a subtraction problem?

M.P.7. Look for and make use of structure. Demonstrate that subtracting a larger value from a smaller value requires the decomposition of the value that is one place value greater. For example, 719 – 646 needs 1 hundred to be decomposed into 10 tens for the subtraction in the tens place, changing the first number to 6 hundreds, 11 tens, and 9 ones. Then, subtract 6 hundreds, 4 tens, and 6 ones to find the difference of 7 tens and 3 ones, or 73. Additionally, represent this decomposition and regrouping using manipulatives such as base-ten blocks.

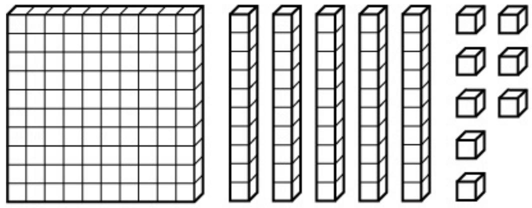
- Ask students to subtract 2 three-digit numbers for which decomposition and regrouping are necessary. For example, given 691 – 435, first decompose each of the three-digit numbers. As part of the decomposition, note that 691 has 1 one while 435 has 5 ones. By decomposing the 9 tens in 691 into 8 tens and 10 ones, the decomposition of 691 is now 6 hundreds, 8 tens, and 11 ones, which makes it possible to subtract 5 ones. After subtracting 4 hundreds, 3 tens, and 5 ones from 6 hundreds, 8 tens, and 11 ones, the difference can be expressed as 2 hundreds, 5 tens, and 6 ones, or 256.

- Ask students to use manipulatives to represent the subtraction of 2 three-digit numbers for which decomposition and regrouping are necessary. For example, given 529 – 371, represent 529 using base-ten blocks as shown.



529

Then, represent the subtraction of 371 by first removing 1 one from the representation of 529. The next step is to remove 7 tens, but there are not 7 tens to be removed. By replacing one of the 5 hundreds with 10 tens, the minuend now has 4 hundred blocks and 12 ten blocks. Now that it is possible, remove 7 tens from the representation of 529 and then remove 3 hundreds. This leaves the difference as shown.



158

Therefore, the difference of 529 and 371 is 158.

Key Academic Terms:

place value, addition, subtraction, three-digit number, properties of operations, compose, decompose, addend, minuend, subtrahend, difference, sum

Additional Resources:

- Lesson: Decoding decomposing {adding 2 four-digit numbers}
- Video: Add & subtract within 1,000 (grade 2)
- Article: Easy strategies for adding and subtracting larger numbers

13

Operations with Numbers: Base Ten
Use place value understanding and properties of operations to add and subtract.
13. Mentally add and subtract 10 or 100 to a given number between 100 and 900.

Guiding Questions with Connections to Mathematical Practices:

How do mental addition and subtraction of 10 or 100 with a given number relate to skip-counting by 10s or 100s?

M.P.2. Reason abstractly and quantitatively. Connect adding or subtracting 10 to skip-counting by 10s. For example, skip-count backward by 10s to show that 865 is 10 less than 875. Additionally, confirm that the number of 10s that are skip-counted indicates the difference in tens.

- Ask students to skip-count by 10s from a given number. For example, give students the number 487 and ask them to skip-count by 10 eight times. Students should say 487, 497, 507, 517, 527, 537, 547, 557, 567. Connect the pattern of skip-counting by 10s to adding 10 by noting that each number in the pattern is 10 more than the previous number.

- Ask students to explain what it means to skip-count by 10s. Help students to conclude that counting involves adding 1 to a number repeatedly and that skip-counting by 10s involves adding 10 to a number repeatedly.

M.P.2. Reason abstractly and quantitatively. Connect adding or subtracting 100 to skip-counting by 100s. For example, skip-count forward by 100s to show that 489 is 100 more than 389. Additionally, confirm that the number of 100s that are skip-counted indicates the difference in hundreds.

- Ask students to skip-count by 100s from a given number. For example, give students the number 722 and ask them to skip-count backward by 100 three times. Students should say 722, 622, 522, 422. Connect the pattern of skip-counting backward by 100s to subtracting 100 by noting that each number in the pattern is 100 less than the previous number.

- Ask students to explain what it means to skip-count by 100s. Help students to conclude that counting involves adding 1 to a number repeatedly and that skip-counting by 100s involves adding 100 to a number repeatedly.

How can mental addition and subtraction of 10 or 100 help to solve problems involving multiples of 10 and 100?

M.P.8. Look for and express regularity in repeated reasoning. Decompose any multiple of 10 or 100 to show repeated addition to solve problems. For example, 279 + 40 can be decomposed to 279 + 10 + 10 + 10 + 10 and can be solved mentally by skip-counting four times by 10 starting at 279 and ending at 319. Additionally, decompose any multiple of 10 or 100 to show repeated subtraction to solve problems.

- Ask students to decompose a multiple of 100 to show repeated addition and use skip-counting to solve the problem. For example, 273 + 500 can be decomposed to 273 + 100 + 100 + 100 + 100 + 100. The value of this expression can be determined by skip-counting by 100s five times, starting at 273. Students should skip-count and say 273, 373, 473, 573, 673, and 773, so the sum 273 + 500 = 773.

- Ask students to decompose a multiple of 10 to show repeated subtraction and use skip-counting to solve the problem. For example, 891 – 60 can be decomposed to 891 – 10 – 10 – 10 – 10 – 10 – 10. The value of this expression can be determined by skip-counting backward by 10s six times, starting at 891. Students should skip-count and say 891, 881, 871, 861, 851, 841, and 831, so the difference 891 – 60 = 831.

Key Academic Terms:

place value, addition, subtraction, more than, less than, multiple, decompose, compose, addend, sum, difference, minuend, subtrahend

Additional Resources:

- Lesson: What is one hundred more? What is one hundred less?
- Worksheet: Add 10 on a number line
- Activity: Subtract 10 and 100
- Worksheet: 100 chart
- Worksheets: Printable number grid

14

Operations with Numbers: Base Ten
Use place value understanding and properties of operations to add and subtract.
14. Explain why addition and subtraction strategies work, using place value and the properties of operations. *Note: Explanations may be supported by drawings or objects.*

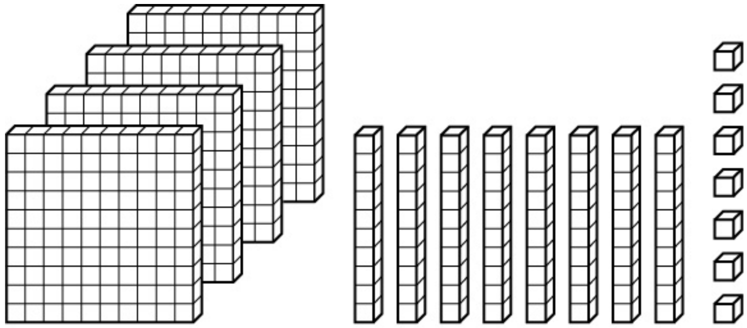
Guiding Questions with Connections to Mathematical Practices:

How can strategies for addition and subtraction be explained in a variety of ways?

M.P.3. Construct viable arguments and critique the reasoning of others. Explain a strategy for an addition or subtraction problem using words, drawings, or objects. For example, explain the thinking that leads to adding hundreds together first, then adding tens together, then adding ones together, and finally combining all three place value groups to find an answer by drawing a model to show why it works. Additionally, demonstrate that a given problem may have multiple effective strategies.

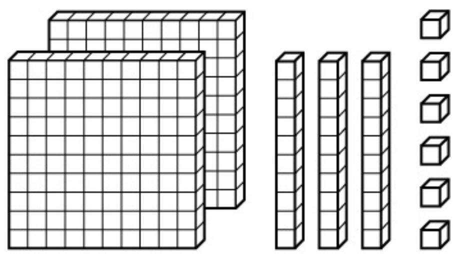
- Ask students to explain strategies for adding 2 three-digit numbers. For example, given the sum $185 + 209$, an appropriate strategy might be to start by adding 5 ones and 9 ones to get 14 ones and then regrouping those 14 ones into 1 ten and 4 ones. After regrouping, the next step would be to add 8 tens (from 185) and 1 ten from the regrouping to get 9 tens. After adding the tens, the next step would be to add 1 hundred and 2 hundreds to get 3 hundreds. The final steps would be to write a three-digit number using the numbers of hundreds, tens, and ones and then to write the equation $185 + 209 = 394$. As an alternate strategy, observe that 209 is one less than 210. Add $185 + 210$ by adding each place value separately. Add 1 hundred and 2 hundreds to get 3 hundreds, add 8 tens and 1 ten to get 9 tens, and add 5 ones and 0 ones to get 5 ones, for a total of 395. Then, subtract 1 from the total to get a sum of $185 + 209 = 394$.

- Ask students to explain strategies for subtracting 2 three-digit numbers. For example, given the difference 487 – 251, lay out base-ten blocks as shown to represent 487.



487

To show the subtraction of 1 one from 7 ones, remove a single one-block from the blocks. To show the subtraction of 5 tens from 8 tens, remove 5 ten-blocks from the blocks. Finally, to show the subtraction of 2 hundreds from 4 hundreds, remove 2 hundred-blocks from the blocks. The remaining blocks are shown.



236

Counting the number of each type of block gives the equation 487 – 251 = 236.

Key Academic Terms:

place value, addition, subtraction, sum, difference, addend, minuend, subtrahend, decompose, compose

Additional Resources:

- Lesson: Adding and subtracting on a hundred chart
- Lesson: Is it “most magically magical”?

15a

Data Analysis
Collect and analyze data and interpret results.
15. Measure lengths of several objects to the nearest whole unit. a. Create a line plot where the horizontal scale is marked off in whole-number units to show the lengths of several measured objects.

Guiding Questions with Connections to Mathematical Practices:

How do line plots compare with rulers and number lines?

M.P.2. Reason abstractly and quantitatively. Explain the similarities between line plots, rulers, and number lines. For example, identify that a line plot is a section of a number line on which measured data are recorded. Additionally, line plots, rulers, and number lines all have different primary uses.

- Ask students to consider that line plots, rulers, and number lines are similar yet different. Help students use these similarities and differences to transfer their understanding of number lines and rulers to line plots. For example, observe that the numbers increase from left to right and that whole numbers are put at equal intervals in all three tools. Further, there may be unlabeled tick marks, but the marks should still be at consistent intervals. Also, the numbers are typically written below the tick marks. Guide students to understand that the differences are primarily in how the lines are used. For example, a line plot is a data display primarily used to show the frequency of a value, a ruler is a tool used to measure distance or the length of an object or space, and a number line is a tool used to show the location or locations of a number or numbers and to solve addition and subtraction problems.

- Ask students to study a list of scenarios and identify if a line plot, a ruler, or a number line is most appropriate to use in each scenario.
    - Allie wants to know if the new scissors she bought will fit inside her pencil box. She knows her pencil box is 9 inches long.

      ruler

    - Jacob has a list of the number of letters his classmates have in their first names. He wants to know how many students have 6 or more letters in their first names.

      line plot

    - Layla wants to buy a new pair of sunglasses that cost $23. So far, she has $16 saved.

      number line

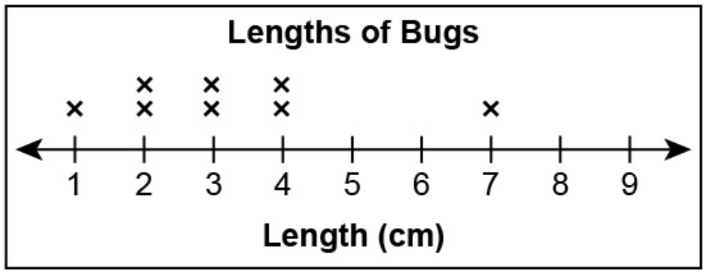
What is a line plot and how can it be used to show measurement data involving whole-number units?

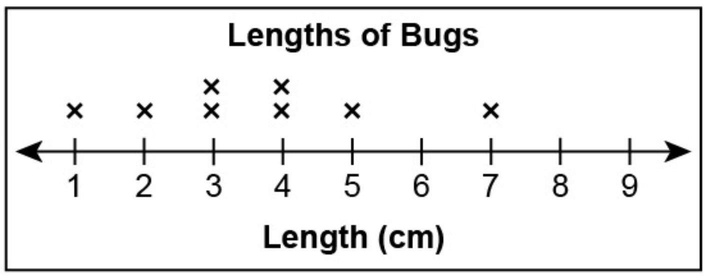
M.P.4. Model with mathematics. Know that a line plot is a graph that displays a distribution of data values (e.g., lengths) such that each data value is marked above a horizontal line with an X. For example, if a box of screws includes 3 screws that are each 1-inch long, 5 screws that are each 2-inches long, and 4 screws that are each 3-inches long, then a line plot can be constructed with three Xs stacked vertically above 1, five Xs stacked vertically above 2, and four Xs stacked vertically above 3. Additionally, a line plot must always have a title, a scale, and a label for the horizontal line.

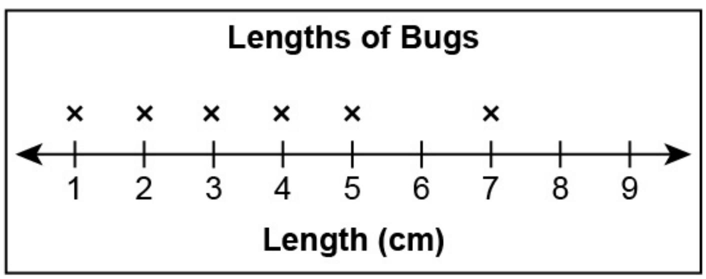
- Ask students to identify which line plot from a series of line plots correctly displays a given set of data. For example, Timothy goes on a bug hunt. He measures and records the length, in centimeters, of each bug he finds. His measurements are shown.

  3, 1, 5, 3, 4 ,4, 7, 2

  Which line plot correctly shows the lengths of all the bugs Timothy finds?

A


B


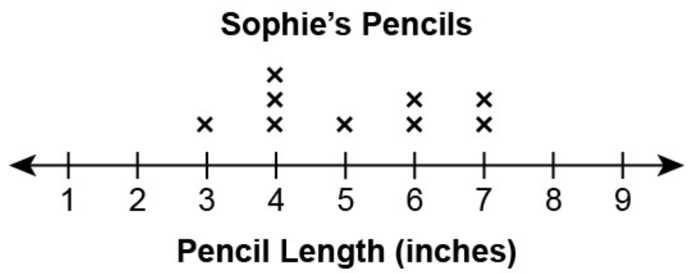
C


  The correct response is B.

- Ask students to use data given in a list to construct a line plot. Remind students to include a title, a scale, and a label for the horizontal line. For example, the lengths, in inches, of Sophie's pencils are shown.

  4, 5, 4, 7, 6, 6, 7, 4, 3

  Ask students to make a line plot to show the lengths of all Sophie's pencils. A possible response is shown.



Key Academic Terms:

data, line plot, whole numbers, horizontal, vertical, title, scale, label, frequency

Additional Resources:

- Article: Creating bar graphs
- Book: Murphy, S. J. (1997). *Lemonade for sale*. New York, NY: HarperCollins. Activity
- Activity: Pencil plot
- Activity: Collecting and representing data
- Game: Fuzz bugs graphing

16a

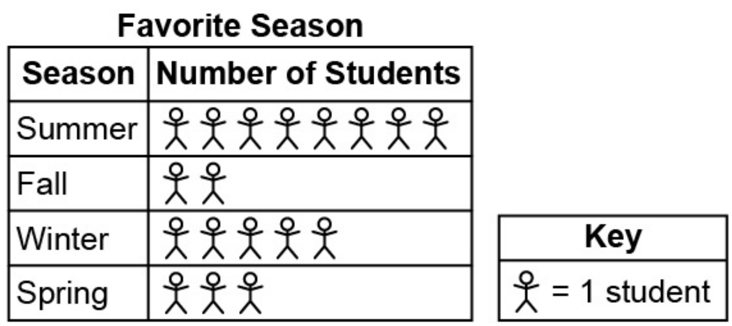
Data Analysis
Collect and analyze data and interpret results.
16. Create a picture graph and bar graph to represent data with up to four categories. a. Using information presented in a bar graph, solve simple “put-together,” “take-apart,” and “compare” problems.

Guiding Questions with Connections to Mathematical Practices:

What is a picture graph and how is it used to represent data?

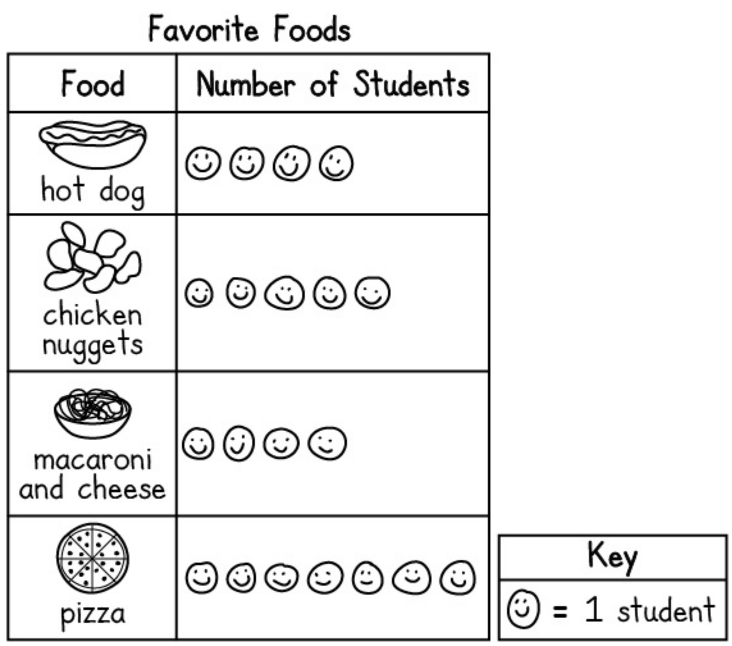
M.P.4. Model with mathematics. Know that a picture graph is a visual representation of data that uses pictures or drawings. For example, if Ben surveys his classmates about their favorite pets and finds that 7 students like dogs, 6 students like cats, and 3 students like birds, then he can construct a graph of the data with pictures of a dog, a cat, a bird, and stick figures. More specifically, a graph with 3 rows and 2 columns can be constructed with the first column showing pictures of a dog, a cat, and a bird and the second column showing 7 stick figures, 6 stick figures, and 3 stick figures next to the corresponding pictures. Additionally, picture graphs include a key to show what the symbols used in the picture graph represent.

- Ask students to use information presented in a list to finish constructing a picture graph that has been started for them. For example, Grayson collects data from each of his classmates regarding which season is their favorite and creates a picture graph.
    - *Summer: 8*
    - *Fall: 2*
    - *Winter: 5*
    - *Spring: 3*

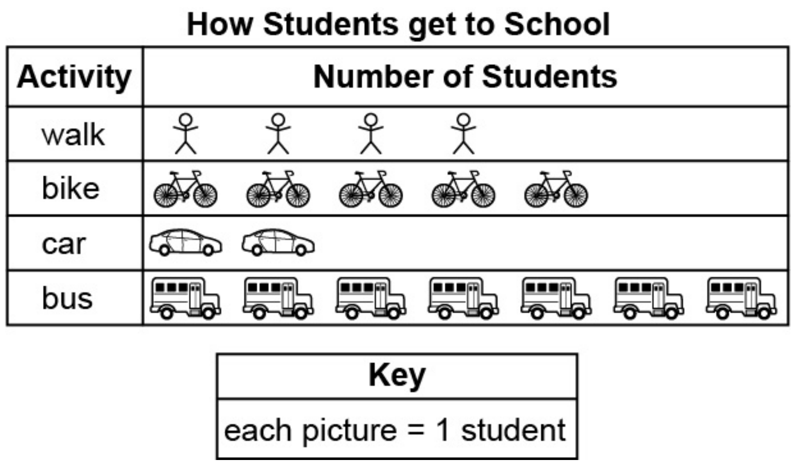


- Survey students in the classroom with a question that has 4 possible answers and record the data so that all students can see the results. Ask students to use the data to construct a picture graph that includes a key. For example, ask students “Which of these foods is your favorite?”
    - Hot Dog
    - Chicken Nuggets
    - Macaroni and Cheese
    - Pizza

  One example of the results of the classroom survey is shown: Hot Dog 4, Chicken Nuggets 5, Macaroni and Cheese 4, Pizza 7.



- Ask students to read information from a picture graph and answer questions about it. For example, show students the following picture graph.

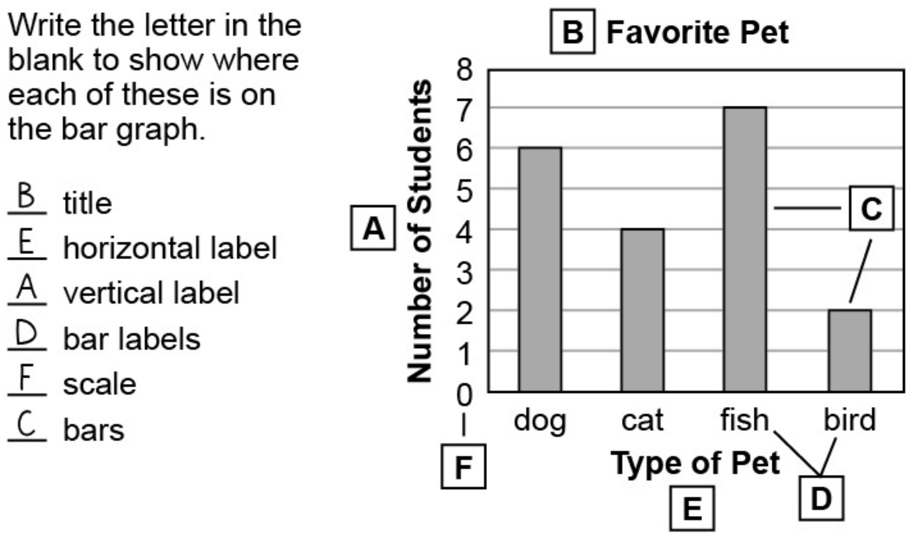


Ask students to use the picture graph to determine how many students use each form of transportation to get to school. In this example, 4 students walk, 5 students bike, 2 students ride in cars, and 7 students ride the bus.

What is a bar graph and how is it used to represent data?

M.P.4. Model with mathematics. Know that a bar graph is a visual representation of data that uses bars of different lengths. For example, if the rainfall in January was 5 inches, the rainfall in February was 4 inches, and the rainfall in March was 6 inches, then a graph can be constructed with the labels "January," "February," and "March" below a horizontal line and a vertical line with 6 equally spaced tick marks labeled from 1 to 6 like a number line. Three bars with heights of 5, 4, and 6 centered above the respective labels "January," "February," and "March" show the amount of rainfall for each of the three months. Additionally, a title, a scale, and labels for both the horizontal and vertical lines must always be included when constructing a bar graph.

- Ask students to look at a completed bar graph that has a title, a scale, labels for both lines, and labels for the bars. Ask students to identify each item on the graph. A possible bar graph with the answers is shown.

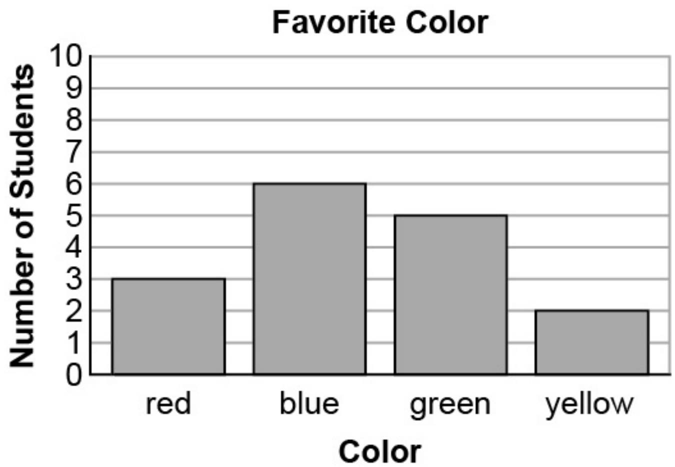


- Ask students to use data given in a table to construct a bar graph displaying the data. Remind students to include a title, a scale, labels for the bars, and labels for both axes. For example, ask students to use the information shown in the table to make a bar graph.

Favorite Color

Color	Number of Students
red	3
blue	6
green	5
yellow	2

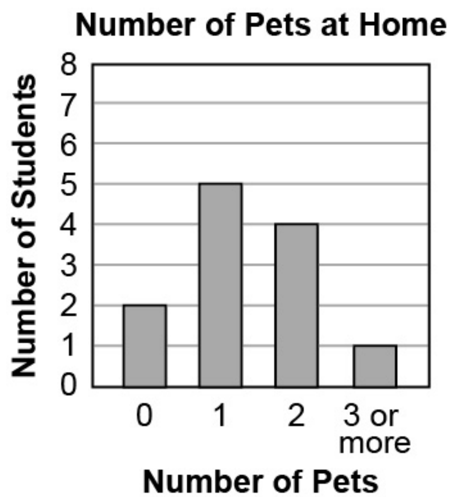
One possible bar graph is shown.



How can the information presented in bar graphs be used to make comparisons?

M.P.4. Model with mathematics. Know that the information presented in bar graphs can be used to show relationships within data. For example, if a bar graph shows that 10 students in a gym class prefer soccer and 8 students prefer kickball, then it can be concluded that 2 more students prefer soccer than prefer kickball. Additionally, bar graphs represent more than/less than comparisons between the different categories of data simply by looking at the height of the bars; the higher the bar, the more frequently that category of data occurred, the shorter the bar, the less frequently that category of data occurred.

- Ask students to use a bar graph to answer questions involving comparing, taking apart, or putting together information from the graph. For example, ask students to use the following bar graph to answer the questions.



  - How many more students have 1 pet at home than have 3 or more pets at home?

    4 students

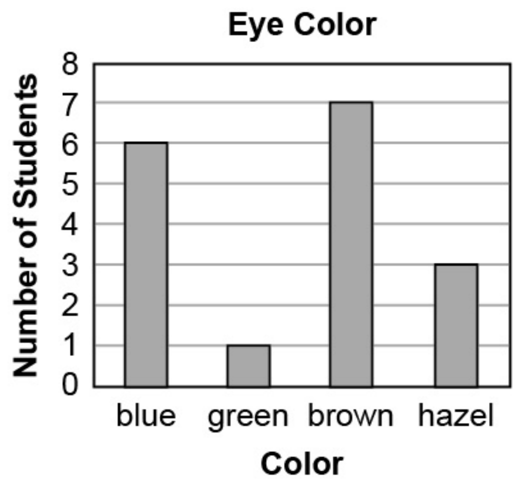
  - How many fewer students have no pets at home than have 2 pets at home?

    2 students

  - How many students have at least 1 pet at home?

    10 students

- Ask students to study a bar graph and write 3 questions that could be answered using only the information shown on the bar graph. When complete, have each student pair with a partner and ask every pair to answer each other's questions. For example, give students the following bar graph and ask them to first study the bar graph, then write three questions that can be answered by using only the information shown in the graph. Some possible student responses are shown.



  - How many more students have blue eyes than green eyes?
  - How many total students have either hazel eyes or green eyes?
  - How many fewer students have blue eyes than brown eyes?

Key Academic Terms:

data set, picture graph, bar graph, scale, column, row, horizontal, vertical, relation, represent, interpret, title, horizontal label, vertical label, bar label

Additional Resources:

- Article: Creating bar graphs
- Book: Murphy, S. J. (1997). *Best vacation ever*. New York, NY: HarperCollins. Activity
- Activity: Collecting and representing data
- Game: Fuzz bugs graphing

16b

Data Analysis
Collect and analyze data and interpret results.
16. Create a picture graph and bar graph to represent data with up to four categories. b. Using Venn diagrams, pictographs, and "yes-no" charts, analyze data to predict an outcome.

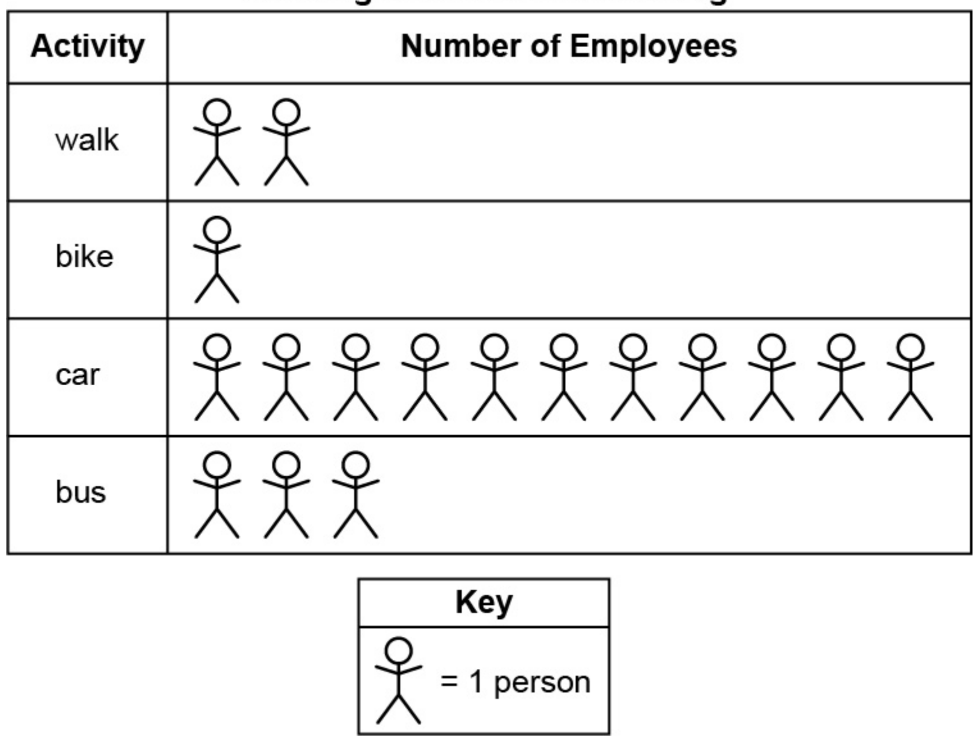
Guiding Questions with Connections to Mathematical Practices:

How can data presented visually be used to make predictions?

M.P.4. Model with Mathematics. Identify that data displayed in graphs and charts can be used to make predictions and inferences. For example, a pictograph shows that of the 25 students in Mr. Jenkins' class on Thursday, 13 students had milk during lunch, 5 students had juice during lunch, and 7 students had lemonade during lunch. Based on this pictograph, students can predict that more of Mr. Jenkins' students will have milk than either juice or lemonade during lunch on Friday. Additionally, illustrate that a Venn diagram can be used to make predictions based on data in which some items belong to more than one category.

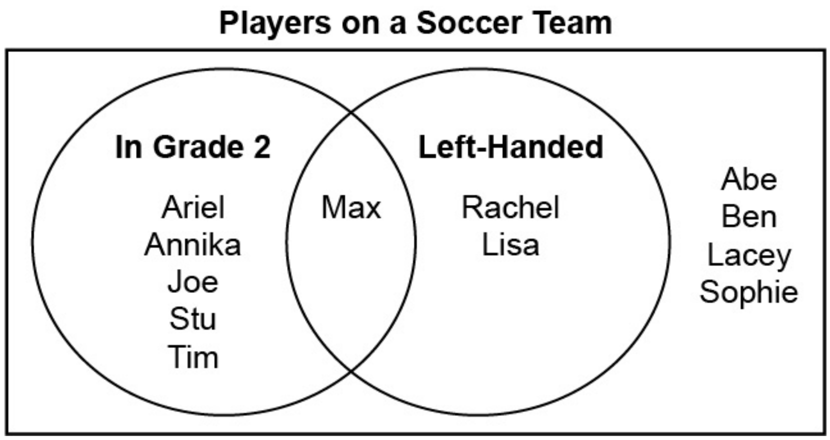
- Ask students to make two or more predictions that are based on data that is displayed in a pictograph. For example, provide students a pictograph that shows how employees arrived at work in the morning.

Arriving at Work in the Morning



Then ask students to study the pictograph and make predictions about how the employees will return home from work in the afternoon. In this case, a student might predict that about half of all the employees will return home from work in a car or that more employees will return home on a bus than a bike.

- Ask students to make a prediction based on data that is displayed in a Venn diagram. For example, provide students with a Venn diagram that shows information about players on a soccer team.



Then ask students to make a prediction about the likelihood that a player with particular characteristics is randomly chosen to be the team captain. In this case, a student could predict that a randomly selected player is not likely to be both left-handed and in grade 2.

How can data presented visually be used to make decisions or assess the fairness of a game or activity?

M.P.3. Construct viable arguments and critique the reasoning of others. Identify that displayed data can be used to make future decisions or to evaluate an activity. For example, a pictograph shows the results of spinning a game spinner 20 times. The spinner has 4 different sections labeled blue, red, green, and yellow. The spinner landed on blue 3 times, red 3 times, green 3 times, and yellow 11 times. Based on the data from the pictograph, students may reasonably conclude that the spinner favors the yellow section more than the blue, red, and green sections or that the yellow section is larger than the other sections. Depending on the game or activity, this conclusion could be used to make and justify decisions. Additionally, emphasize that even when a decision or prediction is based on data, the actual outcome may be different from what is expected.

- Ask students to answer a simple survey question and record the answers in a yes-no chart, and use the chart to make predictions. For example, ask students if they rode a bus to school that day. Students answer yes or no. The chart shown is a record of possible student responses.

Did You Ride the Bus to School Today?

Yes	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓			
No	✓	✓	✓												

Based on the information in the chart, students might predict that a student's answer to this question will most likely be "yes."

- Provide students with a tally chart that shows the results of rolling a number cube. Ask them to explain whether the number cube appears to be fair or not. For example, the following tally chart shows the results of rolling a number cube 18 times.

Rolling a Number Cube

Number	Frequency
⚀	ǀǀ
⚁	ǀǀǀ
⚂	ǀǀ
⚃	ǀǀǀǀ
⚄	ǀǀǀǀ
⚅	ǀǀǀ

In this case, a student might conclude that the number cube is fair because even though it did not land on each side the exact same number of times, none of the sides seemed to be unfairly favored. Notice that if each number occurred the same number of times, then each number would occur three times. The observed occurrence of each number on the number cube is within one of the expected occurrence of three times.

Key Academic Terms:

data set, picture graph, bar graph, scale, column, row, horizontal, vertical, relation, represent, interpret, outcome, prediction, Venn diagram, yes-no chart, pictograph, title, label

Additional Resources:

- Article: Creating bar graphs
- Book: Murphy, S. J. (1997). *Best vacation ever*. New York, NY: HarperCollins. Activity
- Activity: Collecting and representing data
- Game: Fuzz bugs graphing

17

Measurement
Measure and estimate lengths in standard units.
17. Measure the length of an object by selecting and using standard units of measurement shown on rulers, yardsticks, meter sticks, or measuring tapes.

Guiding Questions with Connections to Mathematical Practices:

What is length and how is it measured?

M.P.2. Reason abstractly and quantitatively. Define length as the distance between two points, and identify the appropriate tools for measuring it. For example, the length of a bookshelf (i.e., the distance from one side to the other) can be measured with tools such as a yardstick, a meter stick, or a tape measure. Additionally, certain measurement tools are more appropriate than others when measuring the lengths of objects.

- Ask students to measure the lengths of 3 objects found in their classroom. The first object, such as an unsharpened pencil, should be measured using a ruler. The second object, such as the height of a small table or desk, should be measured with a meter stick. The third object, such as the height of the classroom door, should be measured with a tape measure. Have students record their results in a table that includes the correct unit for each tool.

Measuring Objects

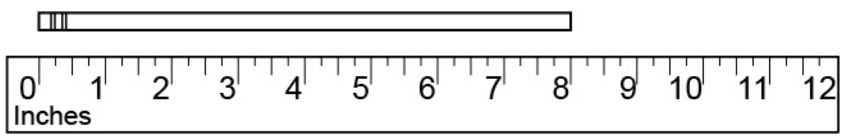
Object	Measuring Tool	Length of Object
pencil	ruler	8 inches
desk	meter stick	1 meter
door	tape measure	78 inches

- Ask students to identify which measurement tool (ruler, yardstick, meter stick, or tape measure) would be most appropriate to measure the length of various objects in the classroom. For example, it is more appropriate to measure the length of a classroom wall using a tape measure than a ruler. Point out that sometimes there is more than one measurement tool that can be appropriate to measure the length of an object, but in other instances, there is one tool that is most appropriate.

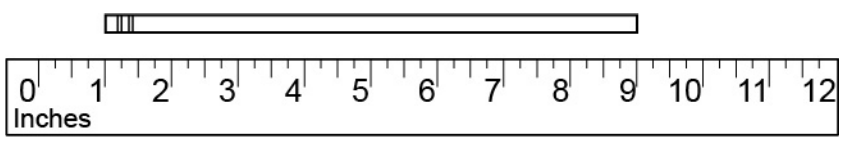
How are measurement tools used?

M.P.5. Use appropriate tools strategically. Demonstrate that when using a ruler or other measurement device, the end marked with "0" is aligned with one edge of the object while the number on the device that is aligned with the other edge of the object indicates the length. For example, the length of a book can be determined as 7 inches if the end of a ruler (US customary) marked with "0" is aligned with one edge of the book and the number "7" on the ruler is aligned with the other edge of the book. Additionally, when the length of an object is between two whole number lengths, the length can be recorded to the nearest inch or half inch.

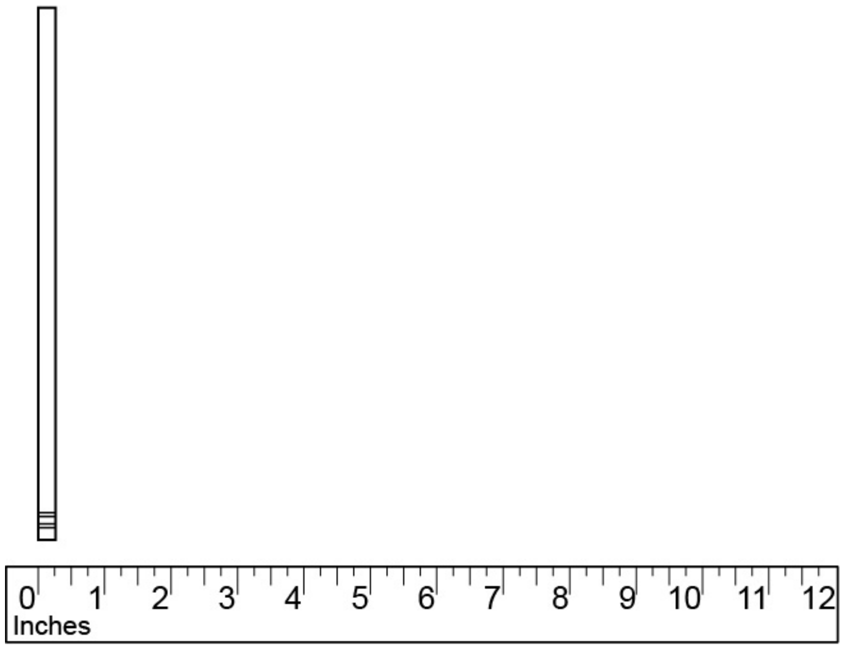
- Model for students how to correctly use a ruler to measure the length of a pencil and record the length.



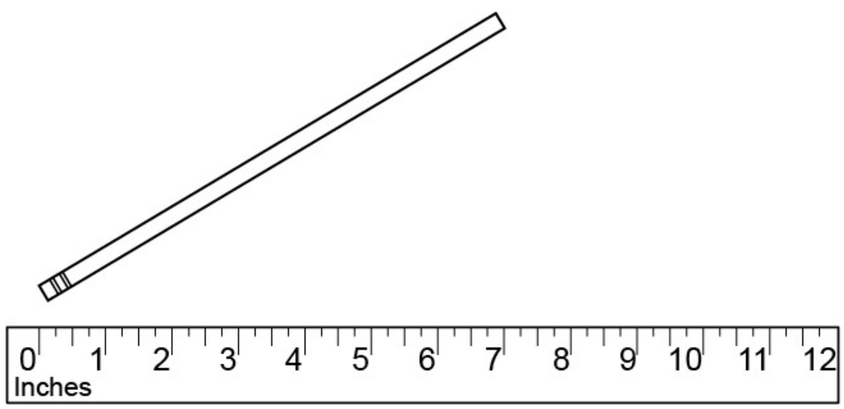
Provide examples of nonconventional ways to use a ruler to measure that same pencil. For example, incorrectly measure the pencil by aligning it to the "1" mark of the ruler and record that result next to the previous result.



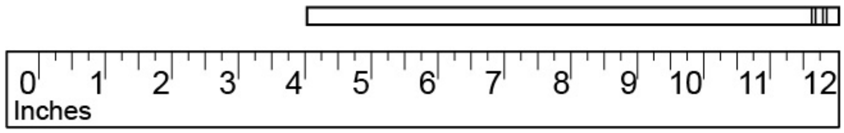
Next, incorrectly measure the length of the pencil by aligning only the eraser at the "0" mark and then having the pencil point vertically instead of lying horizontally along the ruler. Record that result next to the previous results.



Next, align the pencil's eraser at the "0" but have it lie diagonally instead of horizontally across the ruler and record that result.



Lastly, align the pencil at the very right end of the ruler, near the "12" mark, and record that result, which is 4 inches. Ask students to describe how using the last number on the ruler results in an incorrect measurement. Count the units from the beginning of the pencil to the end of the pencil from 4 to 12 and record that result, which is 8 inches. Another way to find the correct measurement of 8 inches is to subtract 4 from 12: 12 – 4 = 8.



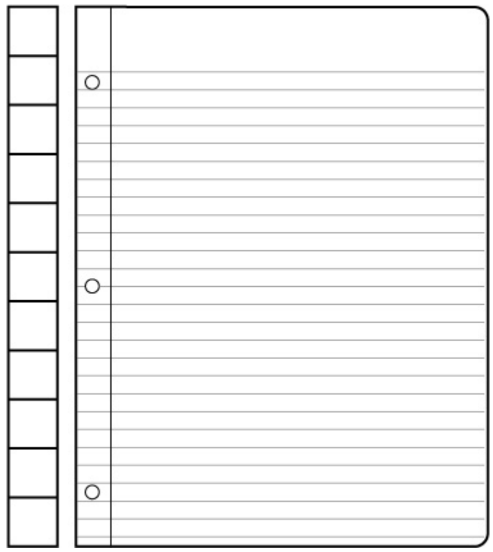
Purposely make errors in front of the students when attempting to measure a set of objects with a meterstick or a yardstick and ask students to identify the errors. Ask students to correctly measure each of the objects and share the correct lengths.

- Explain to students that not all objects will have a length that is a whole number unit. In these cases, estimate the length to the nearest inch or half-inch. Ask students to measure the length of various classroom objects that can be easily measured with an inch ruler but are not exact whole number units long. Have students record the lengths to the nearest inch or half-inch.

What is a unit of length and why is it important?

M.P.6. Attend to precision. Define a unit of length as a measurement that can be an inch, centimeter, or other unit that is of equal size and has no spaces between units and explain the significance of identifying the unit. For example, when measuring an object, choose a homemade ruler made from paperclip lengths with no gaps or overlaps and identify the length unit as a paperclip. Additionally, if the object is 6 paperclips long, then the length is recorded as 6 paperclips.

- Ask students to use colored 1-inch tiles to measure the length of a piece of paper with the tiles aligning at each end of the paper and no gaps between the tiles. Have students record the length of the piece of paper as 11 tiles.



- Ask students to create a unit of length from an object found in the classroom and use it to find the length of another classroom object, perhaps using the length of a textbook to find the length of the teacher's desk. Encourage students to think about which standard measuring unit measures the length of the teacher's desk and, therefore, what an appropriate student-created unit of length could be. For example, it is not very appropriate to use paperclips to measure the length of the teacher's desk. It is more appropriate to instead use textbooks. Students should be recording the length of the object measured with the unit of length they create, such as 8 textbooks, to describe the length of the teacher's desk.

- Tell students that the length of the classroom is approximately 5 (purposely excluding the unit) and then tell students the length of a pencil is approximately 18 (once again, excluding the unit). Ask students how it is possible for the pencil to have a number greater than the number of the classroom wall. Later, explain how the classroom wall measures approximately 5 meters long (with a meter stick) and a pencil measures approximately 18 centimeters (with a ruler). Use this example to discuss the importance of defining and labeling the units of length when measuring objects.

Key Academic Terms:

length, ruler, yardstick, meter stick, measuring tape, inches, feet, centimeters, meters, distance, unit of length

Additional Resources:

- Activity: Comparing lengths in centimeters
- Activity: Estimating meter measures
- Article: Tips for teaching your measurement unit
- Lesson: Measure up!

18

Measurement
Measure and estimate lengths in standard units.
18. Measure objects with two different units, and describe how the two measurements relate to each other and the size of the unit chosen.

Guiding Questions with Connections to Mathematical Practices:

How can an object have different measurements to describe its length?

M.P.2. Reason abstractly and quantitatively. Identify that a unit is a standard of measurement, and that the same length can be measured using different units. For example, the height of a picture frame can be measured using either units of inches or units of centimeters. If the frame is about 11 inches tall, then it is also about 28 centimeters tall. Despite being different, both measurements describe the same distance from one end of the picture frame to the other. Additionally, there are many choices as to which unit of measurement to use when finding the length of an object, but it is important to keep in mind that some units are going to be more appropriate to use than others depending on the size of the object being measured.

- Ask students to recall the different measurement tools that they have used in the classroom (ruler, yardstick, meter stick, tape measures) and have students identify the different units shown on each tool, emphasizing that each unit is used to measure length.

Measurements

Measurement Tool	Units Shown
ruler	inches, centimeters
tape measure	inches, feet, centimeters
meter stick	centimeters, meters
yardstick	inches, feet, yards

Next hold up or point to an object, such as a shoe, and ask students to name at least 2 units that would be appropriate to use to find the length of that object. For example, for a shoe, appropriate units would be inches or centimeters. Repeat this activity with other objects such as the teacher's desk, a paperclip, a textbook, etc.

- Ask students to measure and record lengths, rounded to the nearest whole number, of various objects in the classroom, first in inches and then in centimeters. Ask students to record any general observations they make between the inch measurements and centimeter measurements. Guide students to see that centimeter measurements are greater numerically than inch measurements for each pair.

Measuring Objects

Object	Length in Centimeters	Length in Inches
eraser		
pencil box		
folder		
crayon		

How does the size of the unit impact the measurement of an object?

M.P.2. Reason abstractly and quantitatively. Demonstrate that if the same object is measured with two different units of measurement, then the unit that produces a larger number is smaller than the other unit because there must be more of that unit to cover the same distance. For example, if the length of a rug is first measured as 100 inches and then measured as 254 centimeters, then it can be determined that a centimeter is a smaller unit than an inch because there must be more centimeters than inches to cover the same distance.

- Show students a table of objects and their approximate lengths (to the nearest whole number) and ask students to use just this data (no measuring) to identity which unit of measure is smaller.

Measurements of Objects

Object	Length in Unit 1	Length in Unit 2	Smaller Unit?
marker	6 inches	15 centimeters	centimeters
street light	9 feet	3 meters	feet
shower curtain	2 yards	72 inches	inches
guitar	1 meter	100 centimeters	centimeters
classroom rug	12 feet	4 yards	feet

Key Academic Terms:

unit, length, inches, feet, centimeters, meters, length unit, distance

Additional Resources:

- Book: Myller, R. (1990). *How big is a foot?* New York, NY: Yearling. Activity
- Activity: Measure it twice

19

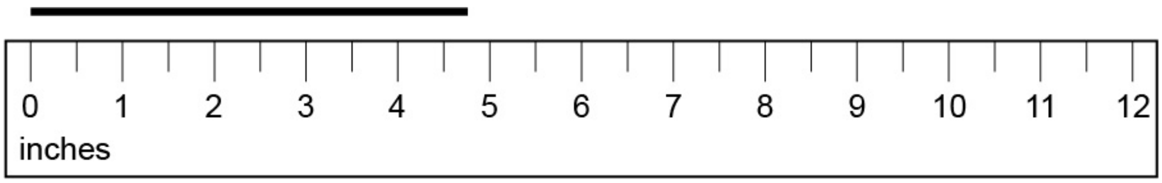
Measurement
Measure and estimate lengths in standard units.
19. Estimate lengths using the following standard units of measurement: inches, feet, centimeters, and meters.

Guiding Questions with Connections to Mathematical Practices:

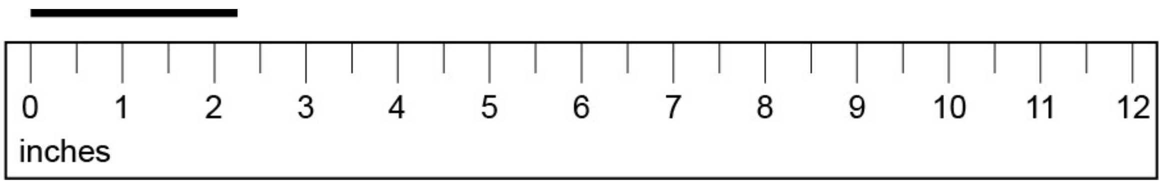
What is a measurement estimate?

M.P.2. Reason abstractly and quantitatively. Know that an estimate is an approximation of measurement rather than an actual measurement. For example, the height of a room that is 8 feet, 3 inches can be estimated to be about 8 feet. Additionally, to estimate the length of an object, determine the whole unit that is nearest to the length of the object.

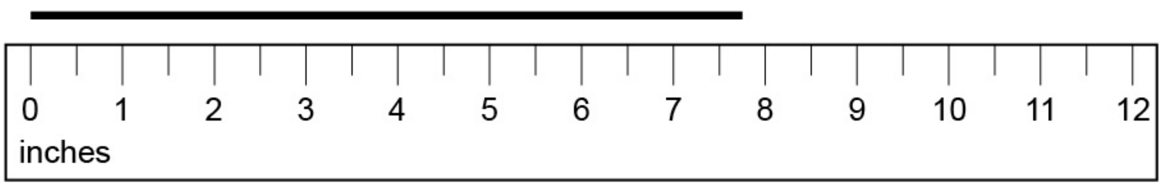
- Present students with a series of lines being measured with an inch ruler and ask them to predict the estimated length of each line to the nearest inch. See the following examples.



5 inches



2 inches



8 inches

- Ask students to use a tape measure to find the lengths of a series of objects that are in between two whole number units. Have students record the exact measurement of each object in feet and inches in one column of a table and write the estimated length, to the nearest foot, in the next column.

Lengths of Objects

Object	Actual Length	Estimated Length
whiteboard	7 feet 4 inches	7 feet
bookcase	3 feet 9 inches	4 feet
locker	5 feet 6 inches	6 feet

How can the reasonableness of an estimate be verified?

M.P.2. Reason abstractly and quantitatively. Connect an estimated measurement to a known measurement to assess the reasonableness of the estimate. For example, if a book is shorter than a piece of paper, and it is known that the piece of paper is 11 inches long, the estimate for the length of the book should be less than 11 inches. Additionally, knowing the exact measurement of one item can help form a reasonable estimate for a second item by reasoning about the relative lengths of the 2 items. For the paper and book example, if the book was only slightly shorter than the paper, then a reasonable estimate may be 9 or 10 inches. Likewise, if the book appeared to be about half the length of the paper, then a reasonable estimate might be 5 or 6 inches.

- Provide students with a series of pairs of objects and a given measurement of one of the objects (from the Object A list) for each pair. Ask students to tell if the second object (from the Object B list) is shorter or longer than the corresponding Object A item. Ask students to use this information to give a reasonable estimate for each of the lengths of items in the Object B list.

Measurements of Objects

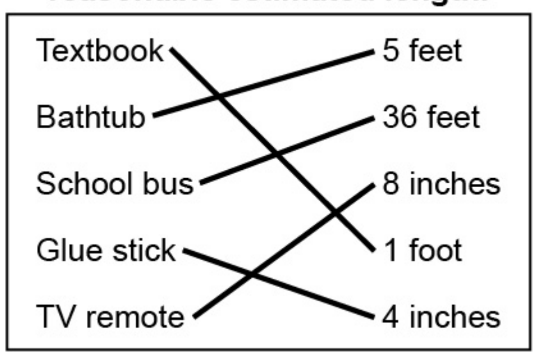
Object A	Object A Length	Object B	Shorter or Longer?	Object B Estimated Length
toothbrush	8 inches	nail clippers	*shorter*	*2 inches*
birthday candle	3 inches	drinking straw	*longer*	*10 inches*
bottle of glue	6 inches	crayon	*shorter*	*4 inches*

- Give students a reference sheet that shows different items and the items' lengths. Ask students to use the information from the sheet to match the objects on the second part of the sheet with a reasonable estimate.

Length Estimates

unsharpened pencil	7 inches
large paperclip	2 inches
teacher's shoe	10 inches
bicycle	6 feet
light post	15 feet

Match each object with the most reasonable estimated length.



M.P.6. Attend to precision. Demonstrate that many estimations can be verified by taking actual measurements. For example, if a person estimates that the height of a mailbox is 45 inches, then a measurement device, such as a tape measure, could be used to determine the actual height. Additionally, having an accurate visualization of the relative size of a unit helps make a more reasonable estimate.

- Ask students to record which unit and measurement tool would be most appropriate to use to find the length of some given objects. Ask students to also give an estimated length of each object. Reveal the actual length of each object and have students compare the estimates to the actual lengths.

Measurements of Objects

Object	Unit	Measurement Tool	Estimated Length	Actual Length	Difference
house key	inches	ruler	3	2	1
computer table	feet	tape measure or yardstick	4	6	2

- Ask students to estimate the length of various classroom objects. Then, have students measure the length of the same objects using an appropriate measurement tool. Have students record any observations made about the estimates. Guide students to discuss the following questions:
    - How close are the estimated lengths to the actual lengths?
    - Are the estimates closer in one unit than another?

What strategies can be used for estimating distances?

M.P.2. Reason abstractly and quantitatively. Extend previous knowledge of measurements of known objects by using a “mental ruler” to make estimations. For example, if it is known that a paperclip is about 1 inch, imagine repeating the paperclip 6 times to measure a dollar bill to estimate the length as 6 inches. Additionally, the closer the item being used to make the “mental ruler” is to the actual length of a unit, the more accurate the estimate should be.

- Ask students to identify at least one classroom object that could be used to represent each of the following: 1 centimeter, 1 inch, 1 foot, and 1 meter. Guide students to select objects that are as close to each of those units as possible. Examples could include: a pencil eraser (about 1 centimeter), a paperclip (about 1 inch), a textbook (about 1 foot), and a pointer stick (about 1 meter). Ask students to measure the selected objects to see how close the representations are to the intended unit. Ask students to find a different object to use for a mental ruler that is not close to the intended unit.
- Ask students to use the selected objects to make and use mental rulers to estimate the lengths of other objects in the classroom. Have students record the estimates in a table and then check the estimates with a standard measuring tool.

Key Academic Terms:

estimate, inches, feet, centimeters, meters

Additional Resources:

- Activity: Comparing lengths in centimeters
- Activity: Estimating meter measures
- Lessons: Grade 2 measurement and data
- Lessons: Measurement—intro to standard units

20

Measurement
Measure and estimate lengths in standard units.
20. Measure to determine how much longer one object is than another, expressing the length difference of the two objects using standard units of length.

Guiding Questions with Connections to Mathematical Practices:

How can the difference in length between two objects be determined?

M.P.5. Use appropriate tools strategically. Measure two objects using the same standard length unit and confirm that the difference in length between the two objects can be found by addition or subtraction. For example, if the length of a pen is measured at 6 inches and the length of an eraser is measured at 2 inches, then the pen is 4 inches longer than the eraser because $6 - 2 = 4$, or it can be said that the eraser is 4 inches shorter than the pen because $2 + 4 = 6$. Additionally, if the difference in length of two objects is known and the length of one of the objects is known, then this information can be used to find the length of the second object.

- Ask students to find the height of a partner using a piece of measuring tape. Record the heights. Complete the following sentence: "_____ is ____ inches taller than _____." For example, if Gabe is 50 inches and Alex is 47 inches, then write: "Gabe is 3 inches taller than Alex."

- Give students a series of pairs of objects to measure using a ruler. Ask students to record the lengths for each pair of objects. Write a subtraction equation and then a comparison statement to show how much shorter or longer one object is than the other.

Measures of Objects

Object A	Length of Object A	Object B	Length of Object B	Equation
pencil	7 inches	crayon	4 inches	7 – 4 = 3 or 4 + 3 =7
glue stick	4 inches	pair of scissors	5 inches	5 – 4 = 1 or 4 + 1 = 5
teacher's shoe	12 inches	student's shoe	9 inches	12 – 9 = 3 or 9 + 3 = 12

Have students make comparison statements. Some examples include the following:

- The pencil is 3 inches longer than the crayon.
- The glue stick is 1 inch shorter than the pair of scissors.
- The teacher's shoe is 3 inches longer than the student's shoe.

- Give students a chart that shows a series of pairs of objects. Give students the length of one of the objects in each pair and then a sentence that tells how much longer or shorter that object is than the other. Ask students to use the given information to find the unknown lengths without measuring.

Measures of Objects

Item A	Length of Item A	Item B	Comparison	Equation	Length of Item B
minivan	17 feet	garbage truck	The minivan is 19 feet shorter than the garbage truck.	17 + 19 = 36	The garbage truck is 36 feet long.
football field	100 yards	basketball court	The football field is 69 yards longer than the basketball court.	100 − 61 = 31 or 69 + 31 = 100	The basketball court is 31 yards long.
couch	84 inches	twin bed	The twin bed is 9 inches shorter than the couch.	84 − 9 = 75 or 9 + 75 = 84	The twin bed is 75 inches long.

Key Academic Terms:

unit, length, difference, comparison

Additional Resources:

- Activity: Pencil plot
- Activity: Gummy worm stretch!

21

Measurement
Relate addition and subtraction to length.
21. Use addition and subtraction within 100 to solve word problems involving same units of length, representing the problem with drawings (such as drawings of rulers) and/or equations with a symbol for the unknown number.

Guiding Questions with Connections to Mathematical Practices:

How can addition and subtraction be used to determine unknown numbers in problems about length?

M.P.2. Reason abstractly and quantitatively. Identify that addition can be used for problems involving the combined length of objects. For example, if Joe stacks 5 blocks that are each 2 inches tall, then the total height of Joe's stack is 10 inches because $2 + 2 + 2 + 2 + 2 = 10$. Additionally, when reporting the sum of the lengths of a series of objects, a correct unit needs to be included such as inches, centimeters, yards, etc.

- Give students a series of real-world problems involving lengths where addition is needed to find the combined length. Ask students to provide the addition equation needed to arrive at the correct sum and remind students to include the correct unit. For example, Betty had her hair cut 3 times last year. The number of inches cut off her hair during each haircut is shown in the table.

Haircuts

Haircut	Number of Inches of Hair Cut
1	4 inches
2	2 inches
3	5 inches

What is the total amount of hair Betty had cut off last year?

The total amount of hair Betty got cut last year was 11 inches.

$4 + 2 + 5 = 11$

Ask students to visually illustrate their solution. For example, give students three pieces of "hair" (yarn) and ask students to illustrate the sum by lining up the pieces of yarn end to end and measuring the combined length. Observe that a ruler can be used as a model in the same manner as a number line.

- Provide students with two different measurements in the same unit. Ask students to write and solve a real-world addition problem using the two lengths. For example, if given the measurements 13 inches and 9 inches, write "Ralph has two pieces of string. One piece of string is 13 inches long and the other piece of string is 9 inches long. What is the total length of string? The total length of string is 22 inches."

M.P.2. Reason abstractly and quantitatively. Identify that addition or subtraction can be used for problems involving the difference of object lengths. For example, if Anna's plant is 60 centimeters tall and 18 centimeters taller than Michael's plant, to find the height of Michael's plant, draw a tape diagram that shows the height of Anna's plant and the difference between Michael's and Anna's plants. Write the equation $60 = \square + 18$, or $60 - 18 = \square$, and find the height of 42 centimeters. Additionally, when determining the difference between the lengths of objects, a correct unit needs to be included such as inches, centimeters, yards, etc.

- Give students a series of real-world problems involving lengths where subtraction is needed to find the difference in the lengths of two objects. Ask students to provide the subtraction equation needed to arrive at the correct difference and remind students to include the correct unit. For example, Hank catches a fish that is 6 inches long. Juan catches a fish that is 9 inches long. How much longer is Juan's fish than Hank's fish? A possible student response is that $9 - 6 = 3$, so Juan's fish is 3 inches longer.

- Provide students with two different measurements in the same unit. Ask students to write and solve a real-world subtraction problem using the two lengths. For example, if given the measurements 48 inches and 53 inches, write "Rachel is 48 inches tall, and Miguel is 53 inches tall. How much taller is Miguel than Rachel? Miguel is 5 inches taller than Rachel."

- Read aloud a series of word problems involving lengths and ask students to identify if addition or subtraction would be needed to solve each problem. Possible student responses are shown after each given word problem.
    - Justin's pencil is 4 inches longer than Adam's pencil. Adam's pencil is 3 inches long. How long is Justin's pencil?

      addition

      $4 + 3 = \square$

    - Two folding tables are set up along a wall. The combined length of the two tables is 14 feet. One of the tables is 6 feet long. How long is the other table?

      subtraction

      $14 - 6 = \square$

    - A teacher is deciding where to put a new bookshelf she purchased for her classroom. The bookshelf is 7 feet long. The wall she wants to put it along is 15 feet long. How much wall space will she have remaining if she places the bookshelf along that wall?

      subtraction

      $15 - 7 = \square$

    - Lacey and Luca measure the lengths of their cars to see how much space the cars will take up in the driveway if parked one in front of the other. Lacey's car is 16 feet long and Luca's car is 14 feet long. What is the total length of their cars?

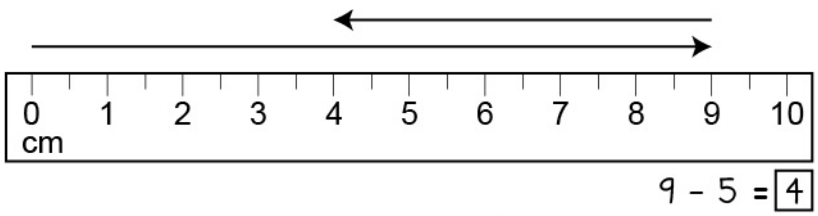
      addition

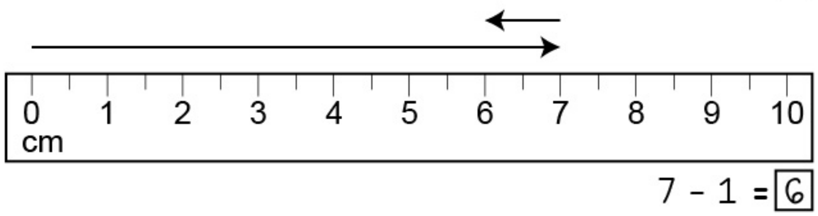
      $14 + 16 = \square$

How can a drawing of a ruler be used to create an equation with an unknown number?

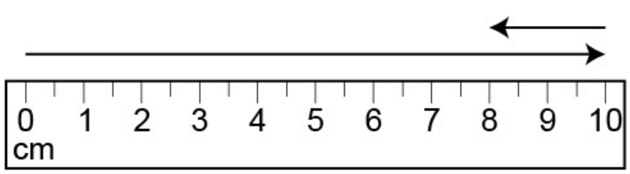
M.P.1. Make sense of problems and persevere in solving them. Demonstrate that finding the difference in length is an additive comparison. For example, the difference between 6 and 2 can be modeled on a number line by placing an arrow of length 6 going to the right from 0 and then placing an arrow of length 2 going to the left from 6. This models the equation 6 – 2, and the difference is 4. Additionally, a ruler can be used in the same way as a number line to model and find the difference in length.

- Ask students to use paper rulers to model subtraction. Ask students to use the rulers to draw a model for various subtraction equations as in the examples shown, which include 9 – 5 = ☐ and 7 – 1 = ☐. Also record the value of the ☐ on each ruler.



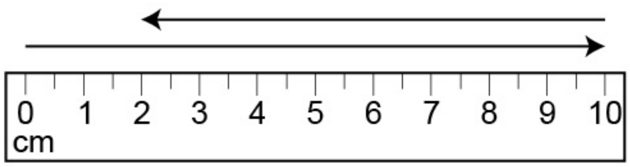


- Ask students to model a word problem by drawing a subtraction equation on a ruler. Each ruler shown models the word problem next to it.



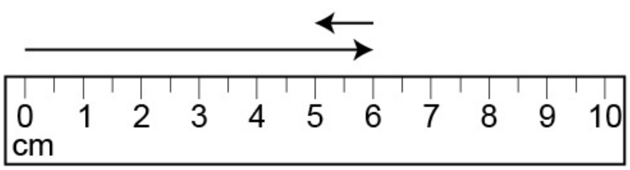
Lily and Kate each have brand new yellow crayons that are 10 cm long. Kate's crayon falls on the ground and part of the tip breaks off. Her crayon is now 2 cm shorter than Lily's crayon. How long is Kate's crayon now?

Kate's crayon is 8 centimeters long.



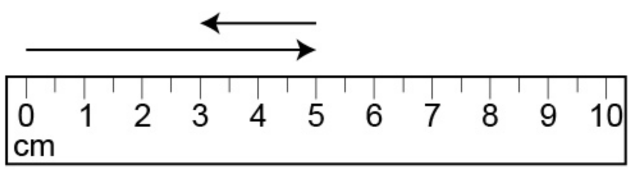
Ahmad's candy bar is 10 cm long and Simeon's candy bar is 8 cm long. How much longer is Ahmad's candy bar than Simeon's candy bar?

Ahmad's candy bar is 2 centimeters longer.



The candles on Ella's birthday cake are all 6 cm long. The candles on Jacob's birthday cake are all 1 cm shorter than Ella's. How long are Jacob's candles?

Jacob's candles are 5 centimeters long.



Parker measures the length of a caterpillar he finds in his yard. The caterpillar is 5 cm long. Later, he finds a beetle that is 2 cm shorter than the caterpillar. How long is the beetle?

The beetle is 3 centimeters long.

Key Academic Terms:

length, addition, subtraction, difference, tape diagram, number line

Additional Resources:

- Video: Solve word problems involving lengths
- Activity: Length word problems

22

Measurement
Relate addition and subtraction to length.
22. Create a number line diagram using whole numbers and use it to represent whole-number sums and differences within 100.

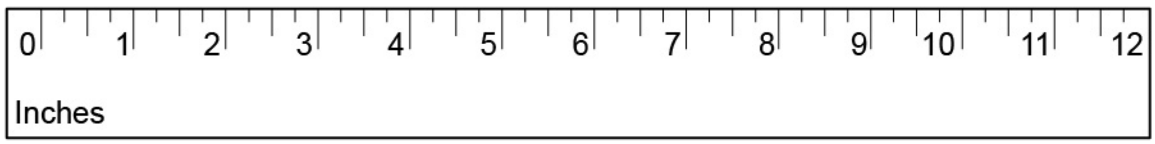
Guiding Questions with Connections to Mathematical Practices:

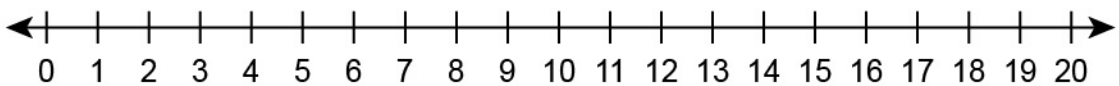
How is a ruler related to a number line?

M.P.2. Reason abstractly and quantitatively. Compare a number line to a ruler and explain the similarities between number lines and rulers. For example, both a number line and a ruler have equally spaced intervals or tick marks that are used to represent distances or lengths. Additionally, in the same way that a ruler can be used to show the length of an object by measuring from one end to the other, a number line can be used to show the distance that exists between two numbers.

- Help students become familiar with using number lines and illustrate how number lines are similar to rulers. Give students number lines and rulers. Ask students to locate a specific number on the ruler by starting at the left-hand side of the ruler and then moving a finger to the right along the number line until reaching the given number. Ask students to use the number line in the same way to locate the same number. Repeat with different numbers.

- Ask students to look at a ruler and a number line to identify as many similarities between the two tools as possible while recording all observations. Guide students to see that both tools are used to show lengths/distances, both tools have numbers that are equally spaced, and both tools have numbers that increase in value when moving from left to right.





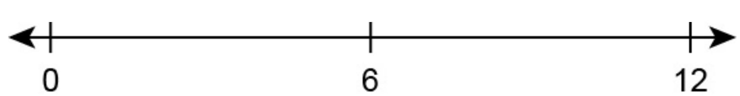
Similarities between a ruler and a number line are listed:

- both show length or distance
- both have numbers that increase in value when moving from left to right
- both have equally spaced numbers

How can the location of a number on a number line be determined?

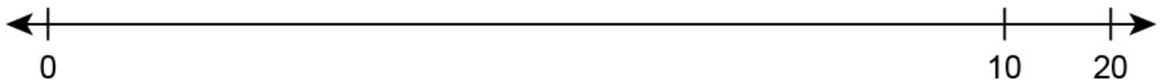
M.P.6. Attend to precision. Accurately approximate the location of a number on a number line by using knowledge of length units and placing marks appropriately when needed. For example, when given an open number line (a line with no tick marks) and the numbers 2, 16, and 20, label the number line 0 on the far left and 20 on the far right; mark the 2 slightly to the right of 0 and mark the 16 slightly to the left of 20. Additionally, each number placed on the number line describes the distance that number is from 0. The greater the number, the farther away from 0 it should be placed.

- Give students an open number line that is exactly 12 inches long with tick marks labeled 0, 6, and 12 correctly placed on the number line. Ask students to label 3 and 9 on the number line. Ask students to use an inch ruler to check the location of the 3 and the 9 that students labeled. Show students that each number along the number line is exactly one inch apart, just like on the ruler.

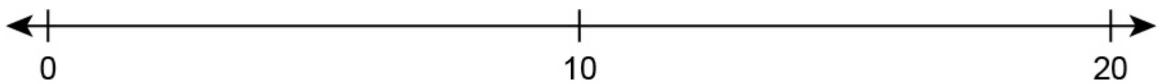


- Give students a series of number lines that have been labeled with tick marks at 0 and 20. Create some number lines with numbers that have been placed and labeled correctly and some lines that have them placed and labeled incorrectly. Ask students to select which number lines are correct. Also ask students to describe how to correct the incorrect number lines.

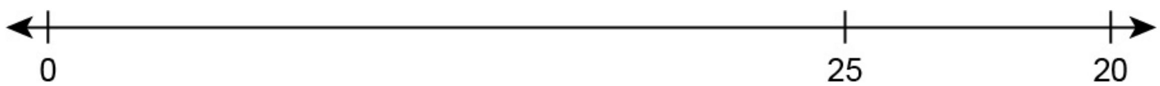
Example 1:



Possible correction:



Example 2:



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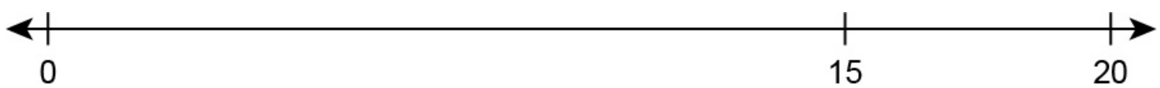
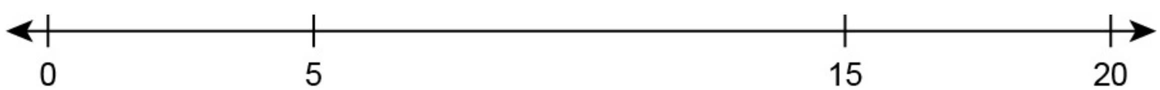
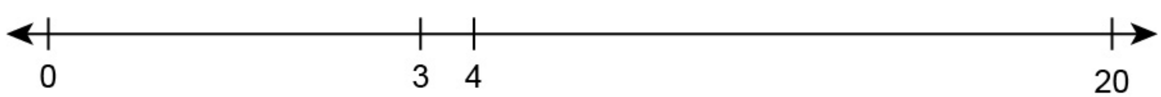


Example 3:



Number line is correct.

Example 4:



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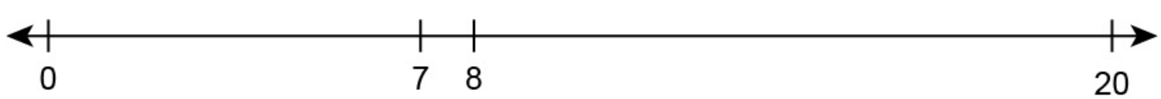
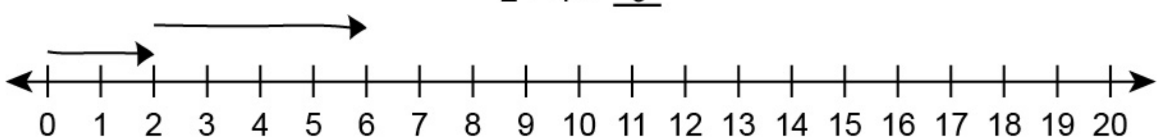


How can the sum of two whole numbers be represented on a number line?

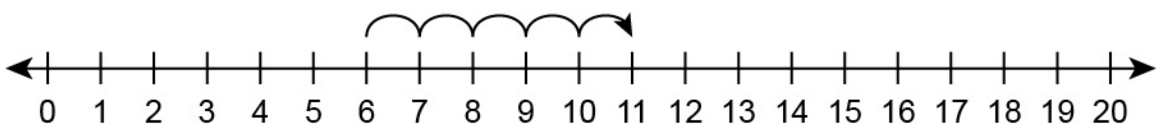
M.P.5. Use appropriate tools strategically. Identify that addition can be represented on a number line by counting forward or to the right. For example, the sum of 10 and 5 can be represented by locating 10 on a number line and then counting 5 tick marks forward or to the right, ending on a point which is located 15 units from 0. Additionally, the sum represents the distance of two combined numbers from 0.

- Ask students to model addition problems on number lines and record the sums. Some examples are shown.

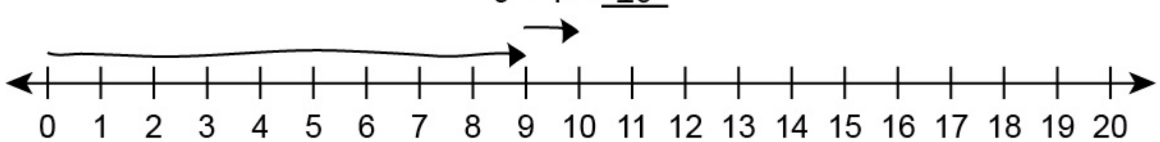
2 + 4 = 6



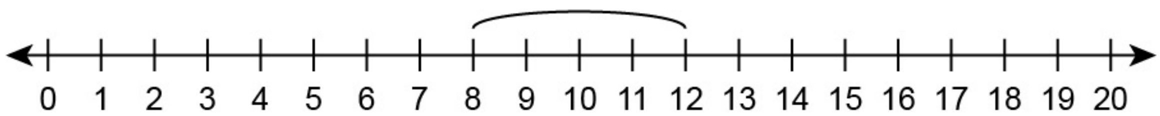
6 + 5 = 11



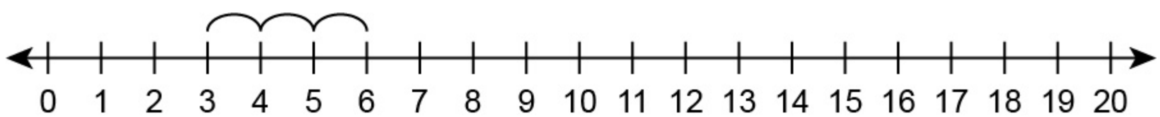
9 + 1 = 10



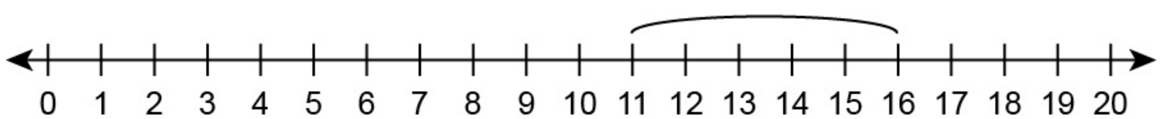
- Give students a series of number lines that model addition problems. Ask students to record the addition problem and sum that is shown on each number line. See the following examples.



8 + 4 = 12



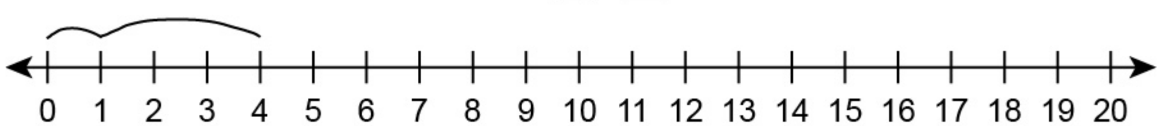
3 + 3 = 6



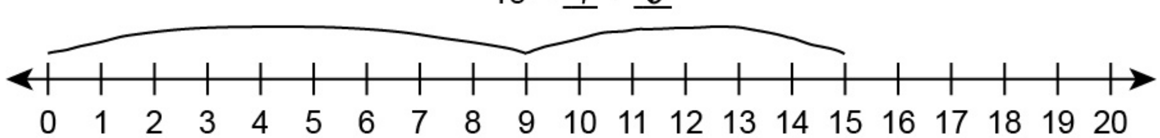
11 + 5 = 16

- Give students a number. Ask students to use the number line to model an addition problem with the given number as the sum. Ask students to record the addition problem for the sum made. Some possible student responses are shown.

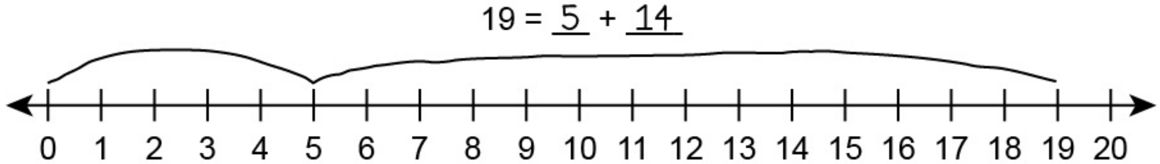
4 = 1 + 3



15 = 9 + 6



19 = 5 + 14

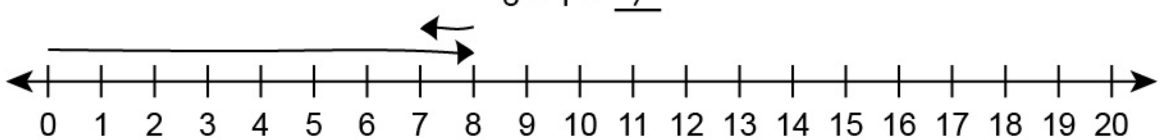


How can the difference of two whole numbers be represented on a number line?

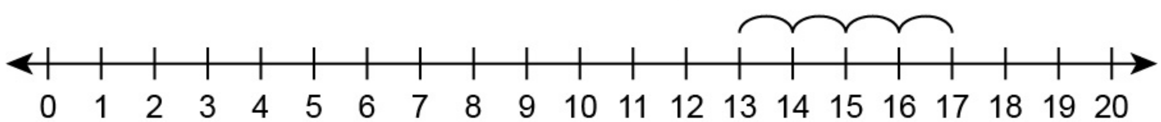
M.P.5. Use appropriate tools strategically. Illustrate that subtraction can be represented on a number line by counting backward or to the left. For example, the difference of 17 and 5 can be represented by locating 17 on a number line and then counting 5 spaces backward or to the left, ending on a point which is located 12 units from 0. Additionally, the difference represents the distance between two numbers.

- Ask students to model subtraction problems on number lines and record the differences. Some examples are shown.

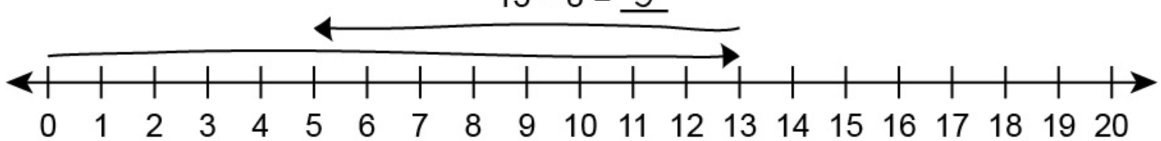
8 – 1 = 7



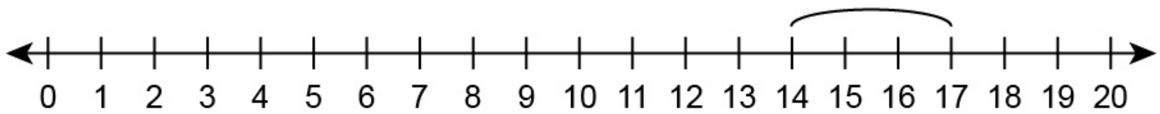
17 – 4 = 13



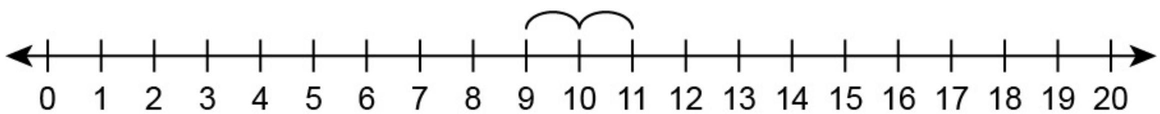
13 – 8 = 5



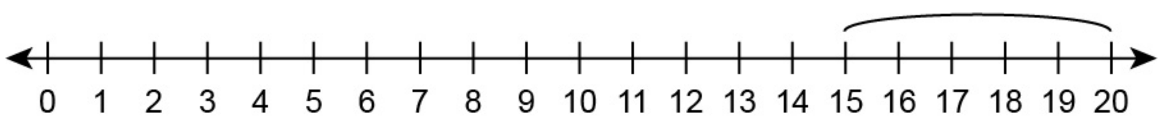
- Give students a series of number lines that model subtraction problems. Ask students to record the subtraction problem and difference that is shown on each number line. See the following examples.



17 – 3 = 14



11 – 2 = 9



20 – 5 = 15

Key Academic Terms:

open number line, sum, difference, whole number, interval, tick marks, length unit

Additional Resources:

- Article: Jump strategy
- Video: Adding and subtracting on an open number line

23a

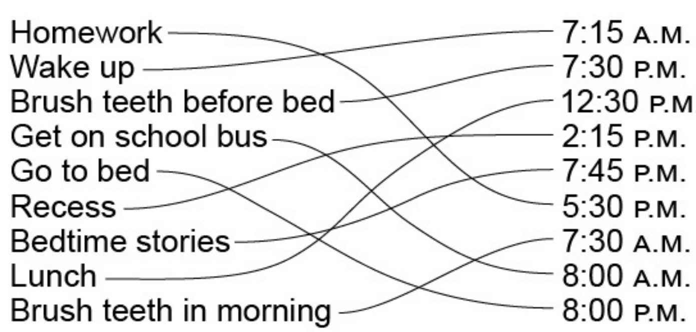
Measurement
Work with time and money.
23. Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m. a. Express an understanding of common terms such as, but not limited to, *quarter past*, *half past*, and *quarter to*.

Guiding Questions with Connections to Mathematical Practices:

What do a.m. and p.m. represent?

M.P.7. Look for and make use of structure. Identify that a.m. designates the time interval between midnight and noon while p.m. designates the time interval between noon and midnight. For example, the time of an activity that occurs in the morning is designated with a.m., while the time of an activity that occurs in the afternoon is designated with p.m. Additionally, 11:59 a.m. is the latest time in the a.m. in a day and 11:59 p.m. is the latest time in the p.m. in a day. A new day always begins at 12:00 a.m. and ends at 11:59 p.m.

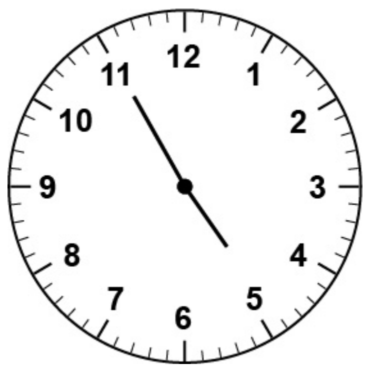
- Give students a variety of routine activities that happen throughout the school day and the time that those activities occur (without including a.m. or p.m.). Ask students to identify if each activity occurs in the a.m. or in the p.m. Some examples, with the student responses, are shown.
    - Lunch: 12:10 *p.m.*
    - Music: 10:25 *a.m.*
    - Bus drop-off: 8:45 *a.m.*
    - Library: 9:05 *a.m.*
    - Recess: 2:25 *p.m.*
    - Math: 11:00 *a.m.*
    - Classroom clean up: 3:35 *p.m.*
- Give students a two-column list. The first column lists daily activities in a nonchronological order and the second column lists times (that include a.m. and p.m.) also in a nonchronological order. Ask students to match each activity with the time that would seem most appropriate for it to occur. An example list is shown.



What is an analog clock and how is it used to tell time?

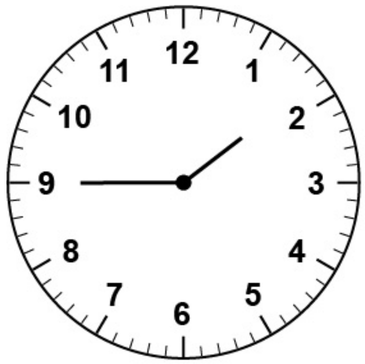
M.P.6. Attend to precision. Identify that an analog clock has moving hands that are used to indicate the hour (short hand) and the minute (long hand) by pointing to different locations around a circle that are labeled 1–12. For example, if the short hand is pointing to 3 and the long hand is pointing to 12, then the clock is showing that the time is either 3:00 a.m. or 3:00 p.m. Additionally, the hands on the clock always move in what is called the clockwise direction, which means starting at the 12 and moving to the right (clockwise) and continuing all the way around the circle.

- Give students an analog clock showing the time 4:55. Ask students to locate various descriptions on the analog clock.

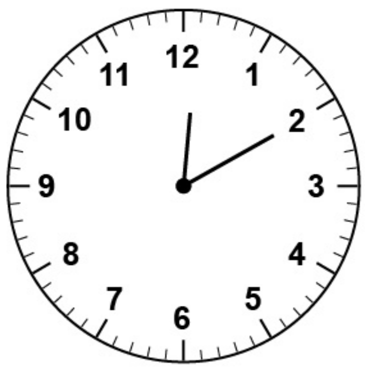


  - Find the hour hand on the clock.
  - Find the minute hand on the clock.
  - Find where the hour hand would point if the hour were 2.
  - Find where the minute hand would point if the minutes were 35.
  - Find where the minute hand would point if the minutes were 50.

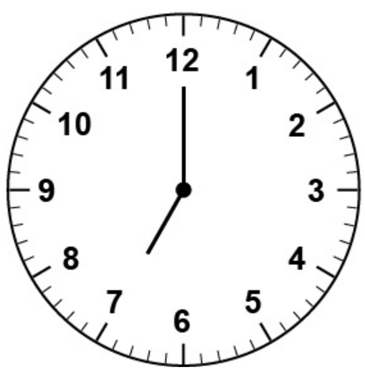
- Ask students to identify the time shown on a series of analog clocks. Some examples are shown.



The time is 1:45.



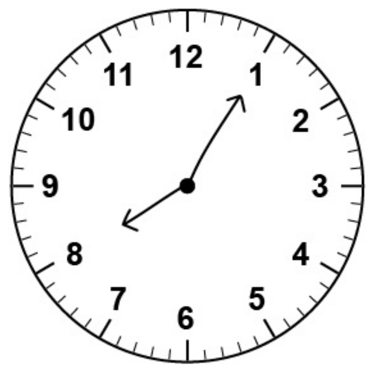
The time is 12:10.



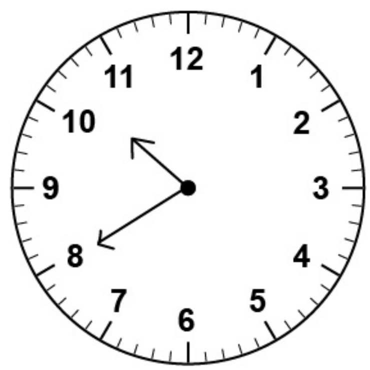
The time is 7:00.

- Give students blank analog clocks and a time written next to each clock. Ask students to show the given time by drawing hands on the accompanying analog clock. Some examples are shown.

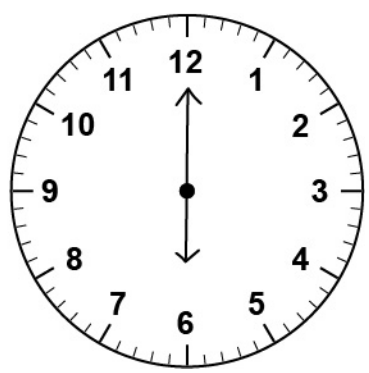
8:05



10:40



6:00



M.P.6. Attend to precision. Identify that there are five marks from each number on a clock to the next number. There is a total of 60 tick marks on a clock, which represent the minutes in each hour. The number 12 at the top of the clock corresponds to 0 minutes. For example, if the short hand is pointing between 2 and 3 and the long hand is pointing to 9, then the time is either 2:45 a.m. or 2:45 p.m. Additionally, when the hour hand is between 2 numbers, the hour is always the lesser of the two numbers. Once it reaches or passes the greater of the two numbers, then that number designates the hour. For example, when the hour hand is between the 8 and the 9, the hour is 8. When it is on the 9 and starts to move past it, then the hour is 9.

- Ask students to give the time given a verbal description of where both the long and short hands on an analog clock are pointing. Some examples are shown.
    - The short hand is pointing between the 4 and the 5 and the long hand is pointing to the 8.

      4:40 a.m. or 4:40 p.m.

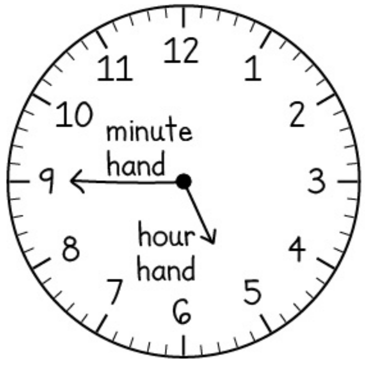
    - The short hand is pointing between the 9 and the 10 and the long hand is pointing to the 1.

      9:05 a.m. or 9:05 p.m.

    - The short hand is pointing between the 3 and 4 and the long hand is pointing to the 3.

      3:15 a.m. or 3:15 p.m.

- Give students a blank copy of an analog clock that only shows the 60 tick marks. Ask students to label the numbers 1–12 in the correct positions to represent each hour shown on an analog clock. Ask students to correctly draw the time 5:45. Lastly, ask students to label which hand is the hour hand and which hand is the minute hand.



On this clock, correctly label the numbers 1–12 to show the hours.

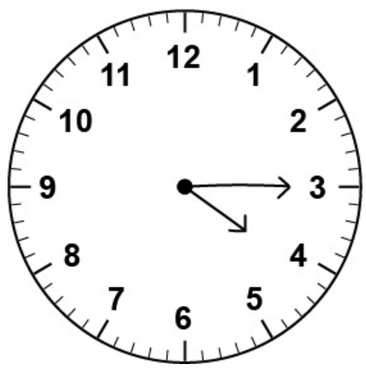
Next, draw the time 5:45.

Lastly, label the hour hand and the minute hand.

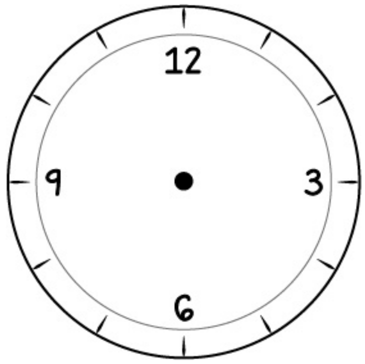
How do fractions relate to expressing the time?

M.P.7. Look for and make use of structure. Connect benchmark fractions to common phrases to express time, such as "quarter past," "half past," and "quarter to." For example, show students a fraction model of a circle with one-half shaded and connect the fraction and the shading to an analog clock showing the minute hand pointing to the 6. Students should know that the fraction is determined by the minute hand, not the hour hand. The 6 is one-half of the way around the circle and can therefore be represented by the fraction one-half. Additionally, fraction models of circles showing one-quarter shaded can be used to apply the phrases "quarter past" and "quarter to."

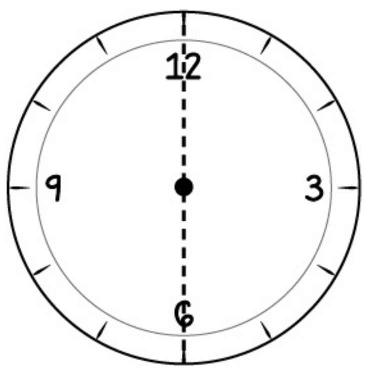
- Provide students with a specific time expressed with the terms "quarter past," "half past," or "quarter to." Ask them to draw the hands on the face of an analog clock. For example, if students are given the time "quarter past four," they should draw a minute hand that points to 3 and an hour hand that is pointing between 4 and 5, as shown.



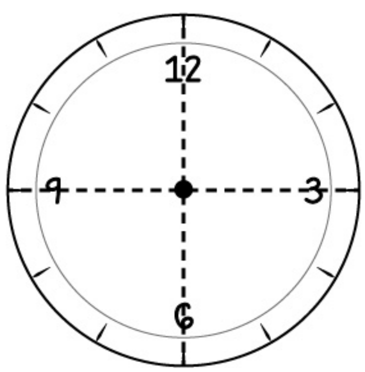
- Give each student a paper plate that has the numbers 12, 3, 6 and 9 located in the correct places to represent a clock face, with tick marks for the other numbers.



Then ask each student to fold the paper plate in half so that a vertical line passes from 12 to 6.



Next, ask students to unfold the plate and fold it in half again so that a horizontal line passes from 9 to 3.



Students should notice that the clock is now divided into 4 quarters. Verbalize different times that are expressed to the quarter or half. Ask students to use the paper plate to determine to which number the minute hand would point.

Key Academic Terms:

analog, digital, a.m., p.m., midnight, noon, morning, afternoon, evening, night, minute hand, hour hand, time interval, tick marks

Additional Resources:

- Article: 5 hands-on ways to teach telling time
- Lessons: Telling time – 2nd grade
- Activity: Time barrier game
- Game: Learn to tell time

24a

Measurement
Work with time and money.
24. Solve problems with money. a. Identify nickels and quarters by name and value.

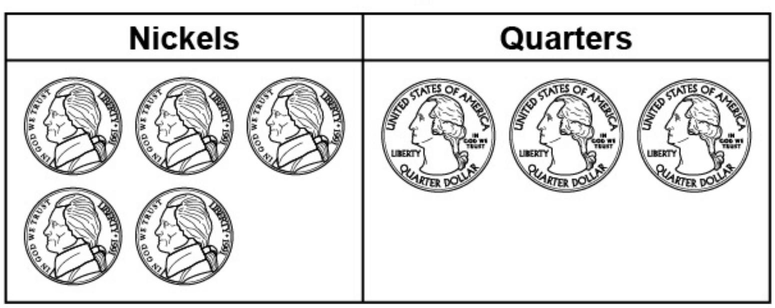
Guiding Questions with Connections to Mathematical Practices:

How is a quarter different from a nickel based on the physical sizes of the coins?

M.P.6. Attend to precision. Determine that when distinguishing a quarter from a nickel, the quarter has the larger size (i.e., a larger diameter) of the two coins. For example, when sorting a pile of coins composed of quarters and nickels, two separate groups can be created based on the two possible sizes. Additionally, know that a nickel is slightly thicker than a quarter and has a smooth edge, whereas a quarter has a ridged edge. Further, know that the likeness of George Washington appears on the quarter while the likeness of Thomas Jefferson appears on the nickel.

- Provide students with a collection of nickels and quarters along with a piece of paper that shows two columns labeled “Nickels” and “Quarters.” Ask students to sort through the collection of coins and place each one in the corresponding column. For example, if the collection of coins includes 5 nickels and 3 quarters, then students would sort the coins accordingly.

Money



- Provide students with a list of incomplete statements about nickels and quarters. Ask them to complete the statements. For example, students complete each statement by identifying the correct word(s).
    - A quarter is ____________________ than a nickel.
      thicker/thinner
    - A ____________________ is wider than a ____________________.
      quarter/nickel quarter/nickel
    - President ______________________________ appears on a nickel.
      Jefferson/Washington

How is a quarter different from a nickel based on the monetary values of the coins?

M.P.2. Reason abstractly and quantitatively. Identify that a nickel represents the monetary value of 5 cents, while a quarter represents the monetary value of 25 cents. For example, when presented with a group of various coins, the coins with a value of 25 cents can be separated and removed from the rest. Likewise, the coins with a value of 5 cents can be separated and removed from the rest. Additionally, determine that the monetary value of a quarter is 5 times as great as the monetary value of a nickel, so 1 quarter can be exchanged with 5 nickels.

- Provide students with different combinations of nickels and quarters. Ask them to identify the monetary value in cents. For example, students could identify that the following collections have values of 20¢, 30¢, and 60¢.

Money

- Provide students with a list of incomplete comparative statements about the monetary value of nickels and quarters. Ask them to complete the statements. For example, students complete each of the following comparative statements.
  - 1 quarter has the same value as _5_ nickel(s).
  - 10 nickels have the same value as _2_ quarter(s).
  - 1 quarter and 1 nickel have the same value as _6_ nickels.
  - _15_ nickels have the same value as 3 quarters.

Key Academic Terms:

cents, quarter, nickel

Additional Resources:

- Video: Counting coins
- Book: Murphy, S. J. (1998). *The penny pot.* New York, NY: HarperCollins. Activity
- Book: Williams, V. B. (1982). *A chair for my mother.* New York, NY: HarperCollins. Activity
- Game: Dolphin dash | Counting money

24b

Measurement
Work with time and money.
24. Solve problems with money. b. Find the value of a collection of quarters, dimes, nickels, and pennies.

Guiding Questions with Connections to Mathematical Practices:

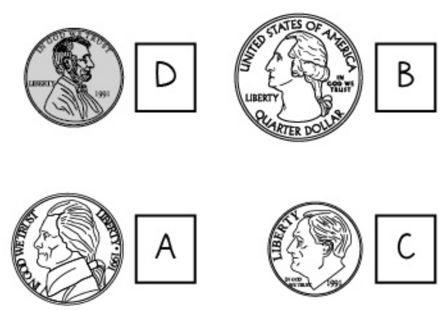
What are the monetary values of quarters, dimes, nickels, and pennies?

M.P.2. Reason abstractly and quantitatively. Know that different coins are used to represent different amounts of money expressed in cents (¢). A quarter represents 25¢, a dime represents 10¢, a nickel represents 5¢, and a penny represents 1¢. Additionally, when two or more coins are combined, the total amount of money can be found by adding the value of each of the given coins together.

- Ask students to match the monetary value with the coin that represents that amount.

Fill the letter in the blank that describes how much each coin is worth.

A. 5 cents
B. 25 cents
C. 10 cents
D. 1 cent



- Ask students to determine the value of a group of like coins. For example, ask students to complete the table shown.

Coin Values

Coins	Total Value
4 pennies	4¢
7 nickels	35¢
6 dimes	60¢
2 quarters	50¢

- Ask students to identify how much total money is present when various coins are combined. For example, ask students to determine the total value of the combinations shown.

41¢

92¢

68¢

How can different coins be used to represent the same amount of money?

M.P.1. Make sense of problems and persevere in solving them. Identify that an amount of money may be represented using various combinations of coins. For example, a quarter is the same amount of money as 5 nickels or 2 dimes and 1 nickel because both groups have a value of 25¢. Additionally, the greater the value of the coin, the fewer of that coin it will take to make a given amount of money. The lesser the value of the coin, the more of that coin it will take to reach a given amount of money.

- Ask students to describe two different combinations of coins that could be used to make a given amount of money. For example, ask students the following questions.
    - What are 2 different combinations of coins that could be used to make 50¢?

      2 quarters or 5 dimes

    - What are 2 different combinations of coins that could be used to make 35¢?

      1 quarter and 1 dime or 3 dimes and 1 nickel

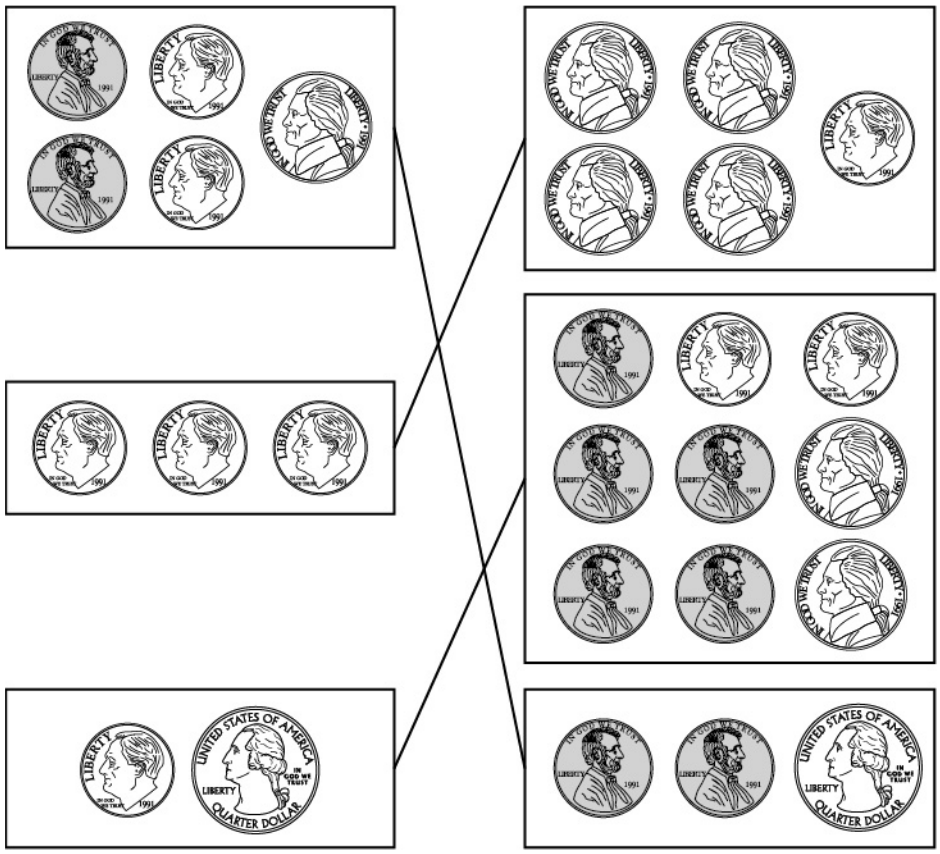
    - What are 2 different combinations of coins that could be used to make 47¢?

      1 quarter, 2 dimes, and 2 pennies

      or

      4 dimes, 1 nickel, and 2 pennies

- Ask students to match the different combinations of coins that have the same value.



- Give students a monetary amount. Ask students to create a combination of coins with that total value using the fewest coins possible. If students are struggling with this concept, start by telling students the number of coins the combination should include. Some examples and possible student responses are shown.
    - 30¢

      1 quarter and 1 nickel
    - 90¢

      3 quarters, 1 dime, and 1 nickel
    - 53¢

      2 quarters and 3 pennies

Key Academic Terms:

dollars, cents, quarter, dime, nickel, penny, dollar bill

Additional Resources:

- Video: Counting coins
- Book: Murphy, S. J. (1998). *The penny pot.* New York, NY: HarperCollins. Activity
- Book: Williams, V. B. (1982). *A chair for my mother.* New York, NY: HarperCollins. Activity
- Game: Dolphin dash | Counting money

24c

Measurement
Work with time and money.
24. Solve problems with money. c. Solve word problems by adding and subtracting within one dollar, using the $ and ¢ symbols appropriately (not including decimal notation). *Example: 24¢ + 26¢ = 50¢*

Guiding Questions with Connections to Mathematical Practices:

What is the relationship between dollars and cents?

M.P.2. Reason abstractly and quantitatively. Know that 1 dollar ($1) is equivalent to 100 cents. For example, if Abby has a dollar bill ($1) and a Bryce has 20 nickels (100¢), then they have the same amount of money. Additionally, whenever monetary values add up to 100¢, that amount can be represented as 1 dollar ($1). For example, if Lily has 72¢ and Thomas has 28¢, their total amount could be represented as 1 dollar ($1) since there are 100 cents in 1 dollar.

- Ask students to solve problems using both 100¢ and $1 to represent 1 dollar. For example, 30¢ + 70¢ = 100¢. Because 100¢ is equal to $1, it is most common to write 30¢ + 70¢ = $1. However, to evaluate $1 – 80¢, it might be helpful to write $1 as 100¢, such as 100¢ – 80¢ = 20¢.

- Ask students to find the total amount of money in problems involving adding or subtracting, using the symbols $ and ¢ appropriately.

    - Logan has a quarter and a dime in his pocket. How much total money, in cents, does he have?

      25¢ + 10¢ = 35¢

    - Kayla has 2 dimes and 5 pennies in her piggy bank. Her brother has 3 quarters in his piggy bank. How much more money, in cents, does her brother have?

      75¢ – 25¢ = 50¢

    - Jacob, Kevin, and Justin are putting their money together to buy a cookie for their friend Adam. Jacob has 3 dimes. Kevin has 2 dimes and 1 nickel. Justin has 1 quarter and 9 pennies. What is the total amount of money, in cents, they have to buy a cookie for Adam?

      30¢ + 20¢ + 5¢ + 25¢ + 9¢ = 89¢

- Give students monetary values. Ask them to describe a combination of coins that equals the given value. Challenge students to do this using the fewest coins possible. For example, given the amounts shown, ask students to find the following combinations.

    - 14¢

      1 dime and 4 pennies

    - 85¢

      3 quarters and 1 dime

    - 55¢

      2 quarters and 1 nickel

Key Academic Terms:

dollars ($), cents (¢), quarter, dime, nickel, penny, dollar bill

Additional Resources:

- Video: Counting coins
- Book: Murphy, S. J. (1998). *The penny pot.* New York, NY: HarperCollins. Activity
- Book: Williams, V. B. (1982). *A chair for my mother.* New York, NY: HarperCollins. Activity
- Game: Dolphin dash | Counting money

25a

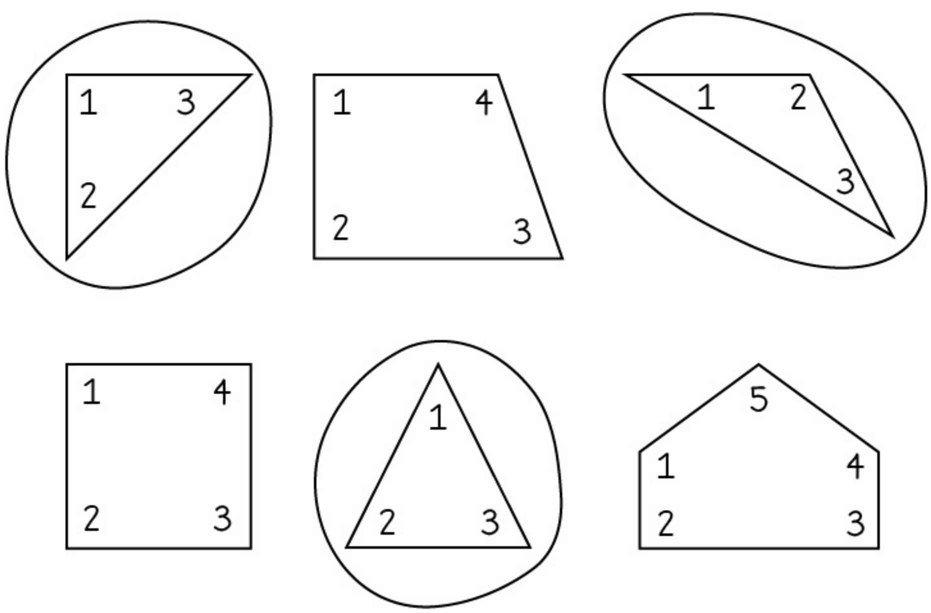
Geometry
Reason with shapes and their attributes.
25. Identify triangles, quadrilaterals, pentagons, hexagons, and cubes. a. Recognize and draw shapes having specified attributes. *Examples: a given number of angles or a given number of equal faces*

Guiding Questions with Connections to Mathematical Practices:

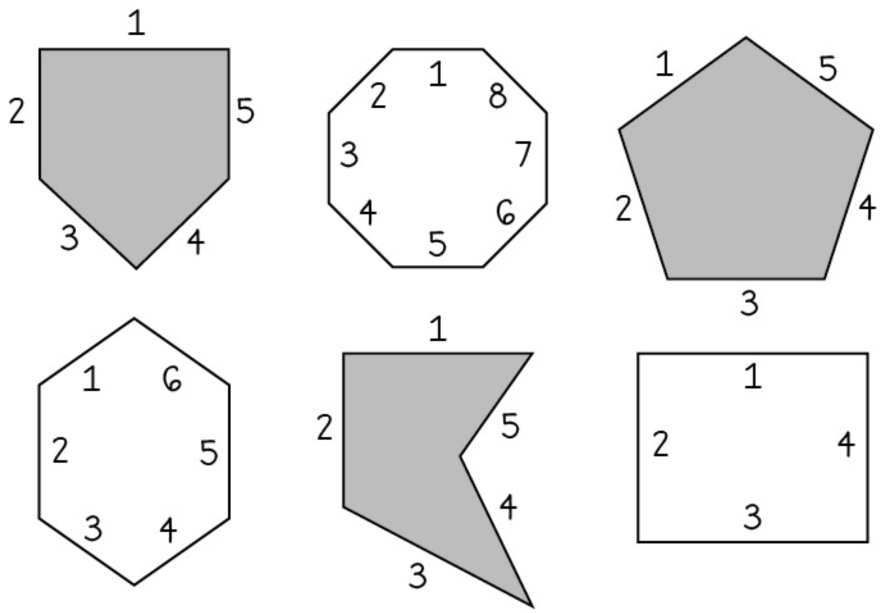
How can shapes be sorted by specified attributes?

M.P.7. Look for and make use of structure. Determine when shapes have the same number of sides, the same number of angles, or other attributes in common, and sort them accordingly. For example, all shapes with 5 sides belong to the group called pentagons. Additionally, examining different attributes may lead to sorting into different groups.

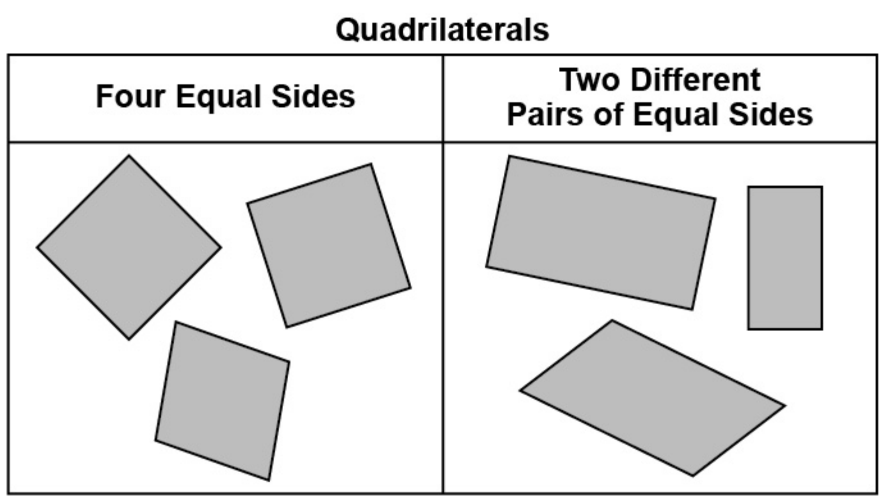
- Ask students to identify all figures in a given set that have a given attribute. For example, identify all figures in a set that have three angles. In the figures shown, students have counted the number of angles in each figure and circled the triangles.



As an additional example, ask students to identify all figures in a given set that have five sides. In the sample shown, students have counted the number of sides on each figure and shaded in the pentagons.



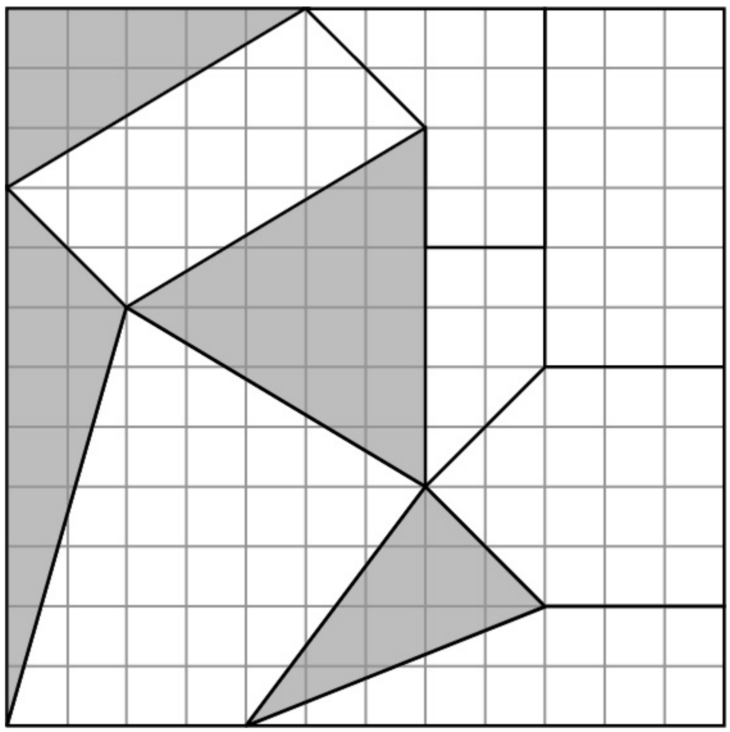
- Ask students to divide a set of figures into two or more categories based on their attributes. For example, identify quadrilaterals that have four equal sides as being in a separate category of quadrilaterals from those that have two different pairs of equal sides. In the example shown, students have separated a set of manipulatives into two categories based on the attributes described.



When is a shape called a triangle, quadrilateral, pentagon, or hexagon?

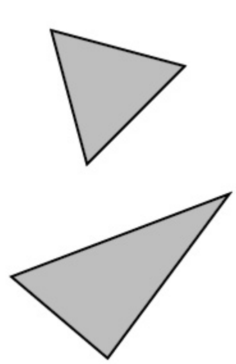
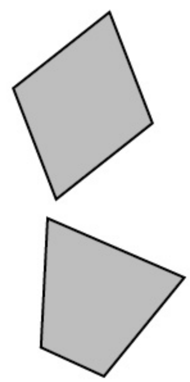
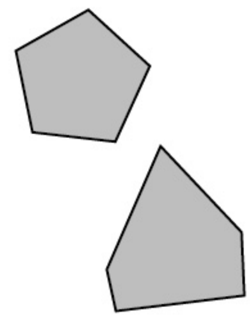
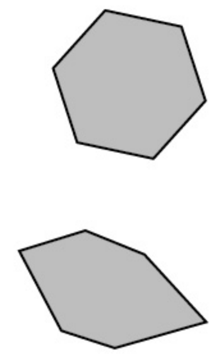
M.P.7. Look for and make use of structure. Observe the number of sides a shape has to determine its classification. For example, a six-sided shape is called a hexagon. Additionally, observe that classifying shapes by the number of angles results in the same classification.

- Ask students to identify figures based on a name. For example, ask students to shade in all triangles in a figure.

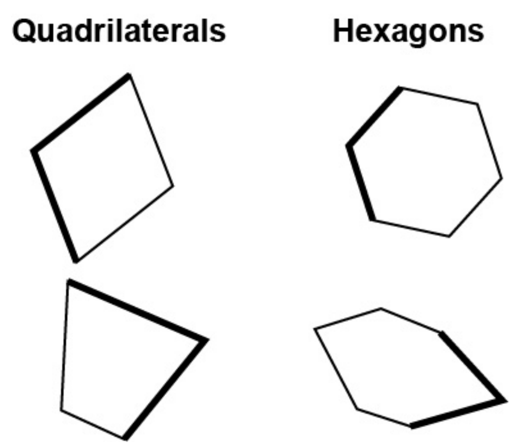


- Ask students to categorize shapes based on a name and explain their reasoning. For example, students place manipulatives in categories based on the number of sides and share their reasoning.

Polygons

Triangle	Quadrilateral	Pentagon	Hexagon
			
All triangles have 3 sides.	All quadrilaterals have 4 sides.	All pentagons have 5 sides.	All hexagons have 6 sides.

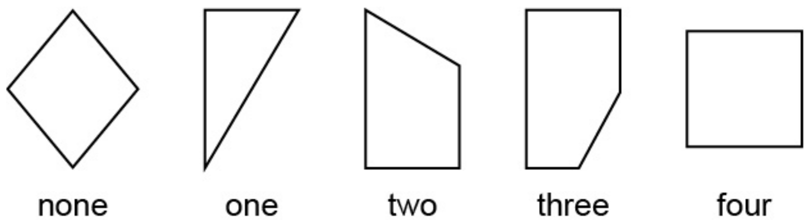
- Ask students to create figures that have the specified attributes. For example, give students a partial image and ask them to complete the image to make a specified shape. In the example shown, students are given the bold lines and draw additional sides to create a quadrilateral or a hexagon.



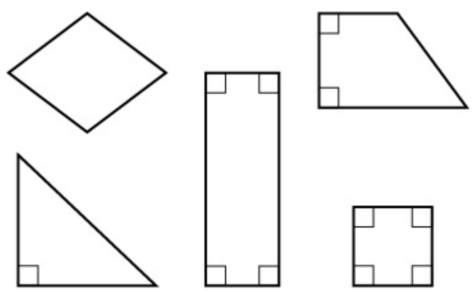
What is needed to determine if an angle is a right angle?

M.P.5. Use appropriate tools strategically. Define a right angle using known objects such as the corner of a piece of paper. For example, find how many right angles are in a quadrilateral by placing an index card in each of the four angles and note how many of the angles line up perfectly with the corner of the index card. Additionally, when a right angle is identified, it can be labeled with a small square inset into the corner.

- Ask students to use a known right angle, such as the corner of a piece of paper, to determine the number of right angles in a figure. For example, the figures shown have been identified as having zero, one, two, three, and or four right angles, respectively.



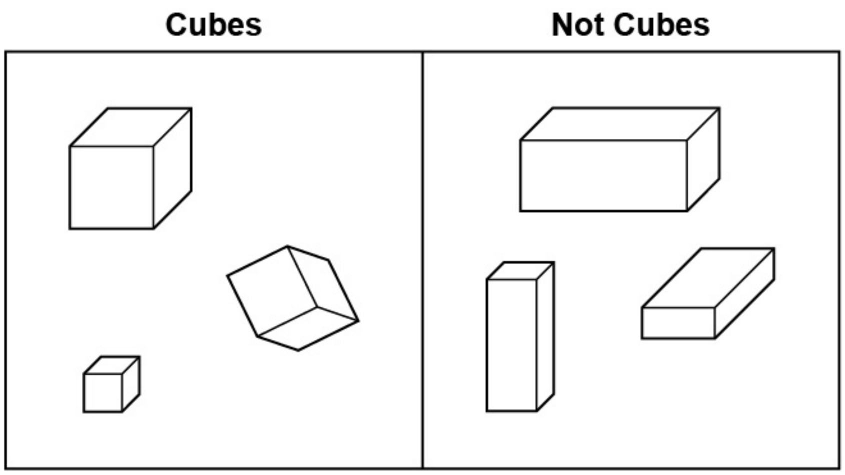
- Ask students to use a known right angle to identify and label right angles in a set of figures. For example, students have labeled the right angles in each figure with small squares inset into each corner.



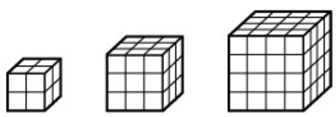
When is a shape called a cube?

M.P.7. Look for and make use of structure. Observe that when a three-dimensional object has six faces, all of which are squares of equal size, it is called a cube. For example, the dice in a board game are cubes. Additionally, not all three-dimensional objects that have one or more square faces are classified as cubes, because six square faces are required.

- Ask students to determine which of a set of 3-dimensional objects is classified as a cube. For example, when given a set of manipulatives, students sort them into cubes and non-cubes.



- Ask students to create cubes using manipulatives. In the example shown, students create three cubes using blocks. Each cube has six equal square faces.



Key Academic Terms:

shape, attribute, angle, face, triangle, quadrilateral, pentagon, hexagon, cube, right angle, side, 2-dimensional, 3-dimensional

Additional Resources:

- Book: Bryant, M.E. (2002). *Shape spotters*. New York, NY: Penguin Young Readers. Activity
- Activity: My shape riddle
- Article: Teaching shapes and their attributes to K–2 students
- Book: Burns, M. (1995). *The greedy triangle*. New York, NY: Scholastic. Activity

26

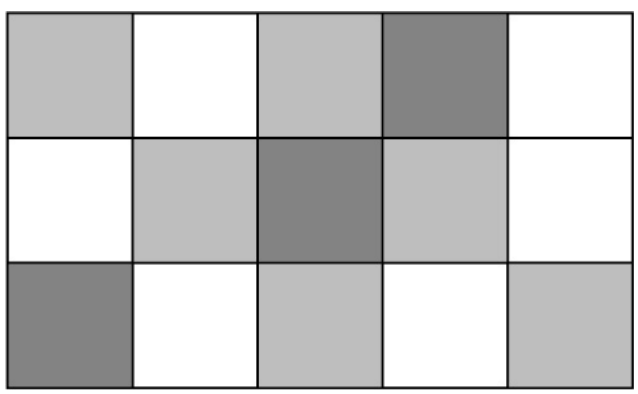
Geometry
Reason with shapes and their attributes.
26. Partition a rectangle into rows and columns of same-size squares, and count to find the total number of squares.

Guiding Questions with Connections to Mathematical Practices:

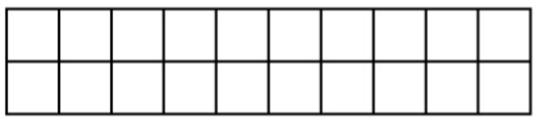
How can a rectangle be decomposed into equal-sized squares?

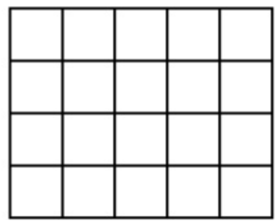
M.P.6. Attend to precision. Construct rows and columns in a rectangle, filling the entire space, to partition a rectangle. For example, use manipulatives of equal-sized square tiles to fill in a rectangle with 2 rows and 6 columns in a structured way and count to determine there are 12 squares. Additionally, use the same 12 squares to create a rectangle with 3 rows and 4 columns or 1 row and 12 columns.

- Ask students to determine the number of square-shaped tiles that are required to fill in a given rectangle and count the resulting numbers of rows and columns. In the example shown, students filled in a rectangle using 15 tiles and counted 3 rows and 5 columns.



- Ask students to construct rectangles with a given number of squares and determine the numbers of rows and columns in the rectangle. For example, given 20 square-shaped tiles, one student may construct a rectangle with 2 rows and 10 columns and another student may construct a rectangle with 4 rows and 5 columns.





Key Academic Terms:

partition, rectangle, square, row, column, decompose

Additional Resources:

- Article: Partition rectangles into rows & columns
- Article: How to teach arrays
- Book: Pinczes, E. J. (1993). *One hundred hungry ants*. Boston, MA: Houghton Mifflin Harcourt Books for Young Readers. Activity
- Activity: Roll a rectangular array

27a

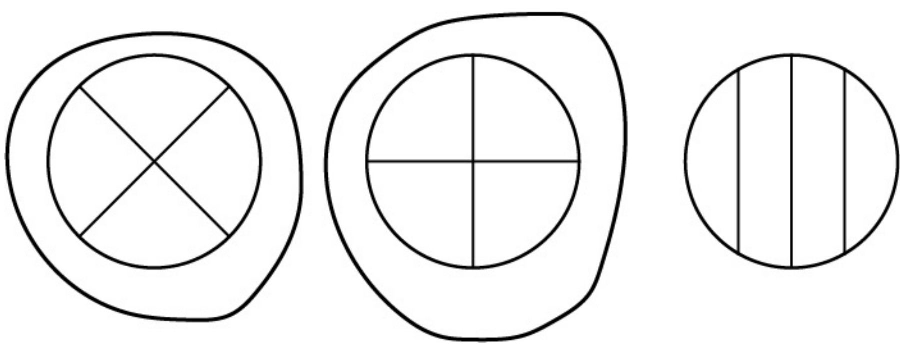
Geometry
Reason with shapes and their attributes.
27. Partition circles and rectangles into two, three, or four equal shares. Describe the shares using such terms as *halves*, *thirds*, *half of*, or *a third of*, and describe the whole as *two halves*, *three thirds*, or *four fourths*. a. Explain that equal shares of identical wholes need not have the same shape.

Guiding Questions with Connections to Mathematical Practices:

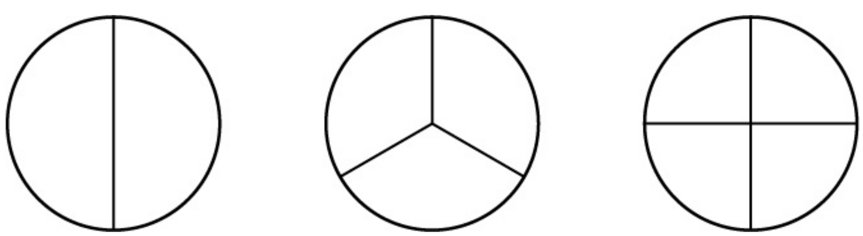
How can circles be partitioned into equal shares?

M.P.6. Attend to precision. Decompose a circle into equal-sized slices, all coming together at the center. For example, think of a circular pizza that has been sliced in half horizontally and then sliced in half vertically to make 4 equal-sized slices. Additionally, identify when a circle has been partitioned into equal areas.

- Ask students to identify circles that have been decomposed into equal-sized slices. In the example shown, the circles that are partitioned into equal shares have been circled.



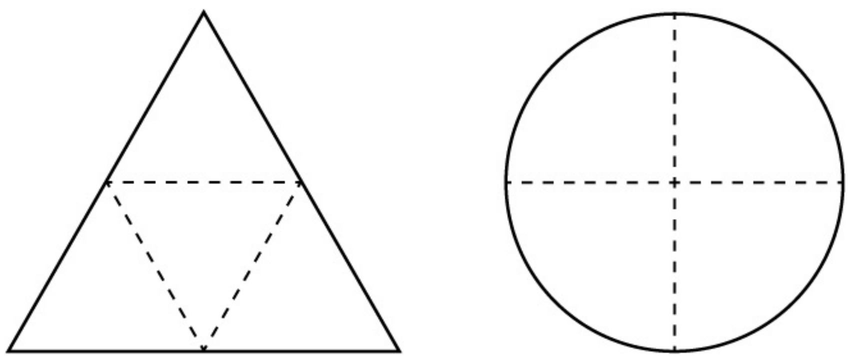
- Ask students to partition a circle into equal shares. In the example shown, one circle has been correctly partitioned into halves, one into thirds, and another into fourths.



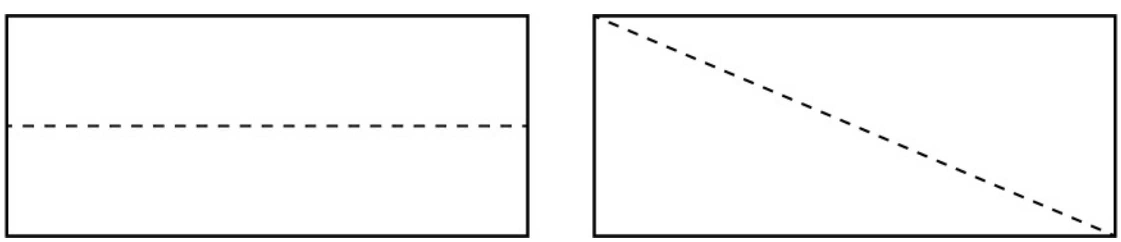
How are rectangles and circles partitioned the same, and how are they different?

M.P.2. Reason abstractly and quantitatively. Verify that rectangles and circles can both be partitioned into equal shares, but the shapes of those partitions will be different. For example, a rectangle partitioned into 2 equal shares may be composed of 2 rectangles, while a circle partitioned into 2 equal shares will be composed of 2 shapes that each have one straight side and one rounded side. Additionally, a square can be partitioned into 4 equal shares that have 3 straight sides by connecting opposite corners, while a circle sliced into 4 equal shares will be composed of shapes with two straight sides and one rounded side.

- Ask students to describe the shapes created by partitioning figures into equal shares. For example, the triangle shown has been partitioned into four equal shares that are also triangles, and the circle has been partitioned into four equal parts that are not also circles.



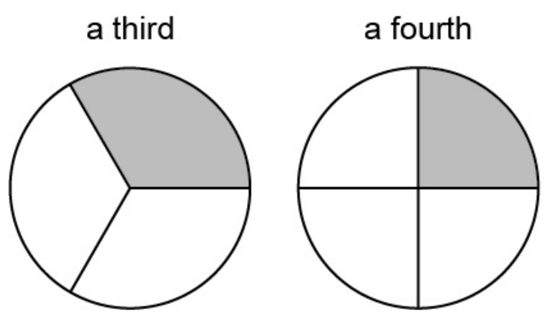
- Ask students to partition a figure into equal shares of the same and of different shapes. For example, the rectangle on the left has been partitioned into two equal shares, both of which have four straight sides, while the rectangle on the right has been partitioned into two equal shares, both of which have three straight sides.



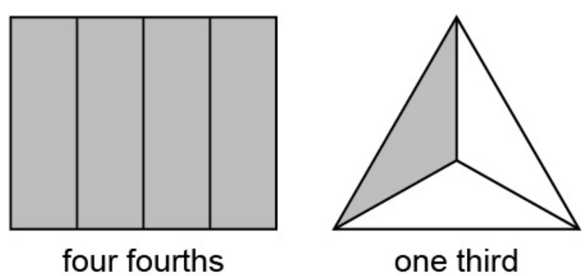
How can partitions of shapes be described?

M.P.7. Look for and make use of structure. Describe partitions of a shape with words that indicate the parts of the whole shape: there are two halves, or three thirds, or four fourths. If only one of the partitions is being described, use fractional words: half of, a third of, a fourth of, etc. For example, a rectangle that is partitioned into three equal shares is composed of three thirds, and one of those shares is a third of the whole rectangle. Additionally, a circle that is composed of two equal shapes is partitioned into two halves, each of which is one half of the circle.

- Ask students to describe one part in a figure that has been partitioned into equal shares. In the example shown, the circle on the left has been partitioned into three equal parts, one of which is shaded and called “a third,” while the circle on the right has been partitioned into four equal parts, one of which is shaded and called “a fourth.”



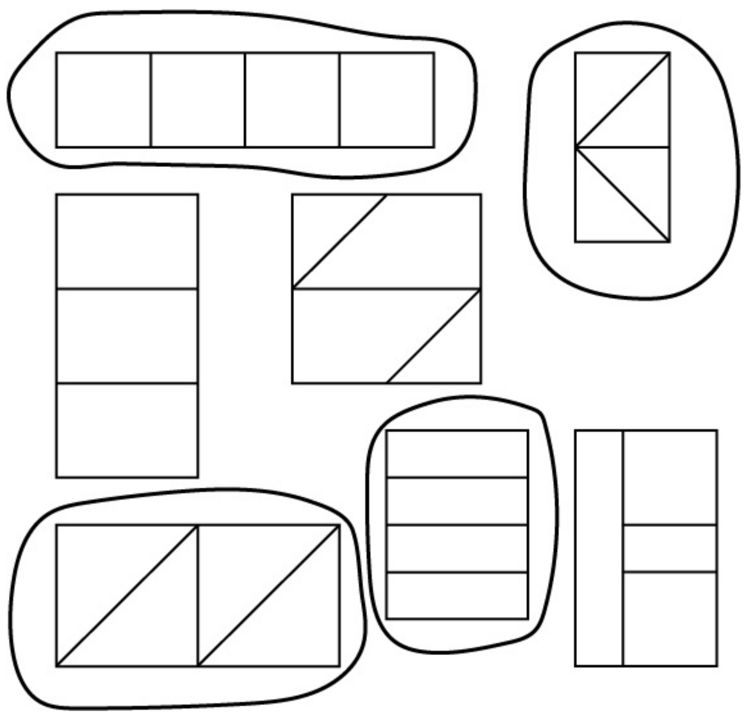
- Ask students to identify one or more parts of a whole in a figure that has been partitioned into equal shares. For example, a rectangle partitioned into four equal parts with all four parts shaded represents “four fourths” of the whole, and a triangle partitioned into three equal parts with one part shaded represents “one third” of the whole.



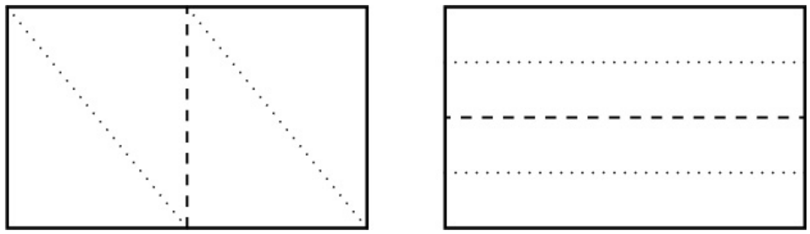
How can equal shares of identical rectangles be represented in different ways?

M.P.3. Construct viable arguments and critique the reasoning of others. Partition a rectangle in multiple ways to see that there are different ways to make equal shares of the same size that may not be the same shape. For example, a square can be partitioned into fourths by drawing a line down the middle vertically and then another line through the middle horizontally to make 4 smaller square equal shares, or it can have a line down the middle vertically and then two more vertical lines on either side of the first line, splitting the two halves in half to make 4 thin rectangular equal shares, or it can be partitioned diagonally to make 4 equal triangular shares. Additionally, any two halves of a partitioned figure can then again each be divided into two equal parts to create four equal-sized pieces.

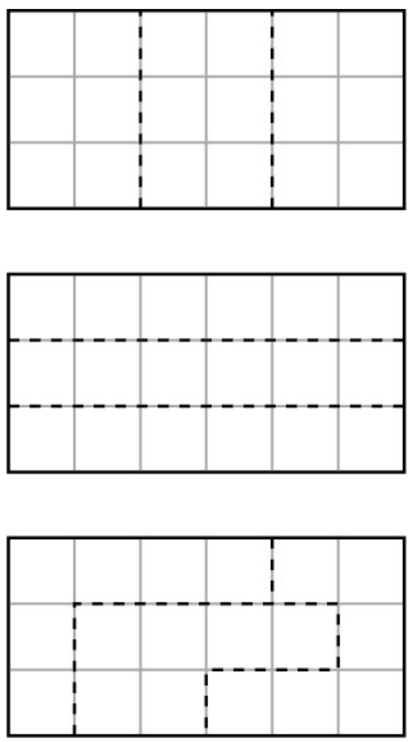
- Ask students to identify rectangles that are partitioned into fourths and those that are not. In the example shown, rectangles that are partitioned into fourths are circled.



- Ask students to partition rectangles into fourths by first partitioning the whole into two parts and then partitioning each half into two parts. In the examples shown, each rectangle is partitioned with a dashed line, and each half is then partitioned with a dotted line. The partitioning can be done by folding a piece of paper, so the dashed lines and dotted lines show where the folds should be made.



- Ask students to decompose rectangles into three equal shares in multiple ways. In the example shown, three rectangles have each been decomposed into thirds.



Key Academic Terms:

partition, circle, rectangle, equal shares, halves, thirds, fourths, half of, third of, fourth of, whole, vertical, horizontal, diagonal, decompose

Additional Resources:

- Lesson: Partitioning the whole into equal shares
- Activity: Geoboard halves
- Activity: Fraction barrier game
- Video: Partitioning circles and rectangles

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